File S1: OxCal age-depth model input.

```
Plot()
 {
 Outlier_Model("General",T(5),U(0,4),"t");
 P_Sequence("",1,0.3,U(-2,2))
 {
  Boundary();
  Curve("Atmospheric","IntCal20.14c");
  R_Date("348.5 – 351",12690,150)
  {
   z=350;
   Outlier(0.05);
  };
  R_Date("303-304",9875,35)
  {
   z=303.5;
   Outlier(0.05);
  };
  R_Date("233.5-235.5",8270,35)
  {
   z=234.5;
   Outlier(0.05);
  };
  R_Date("197-199",6485,25)
  {
   z=198;
   Outlier(0.05);
  };
  R_Date("138-140",5085,20)
  {
   z=139;
   Outlier(0.05);
  };
  C_Date("Aniakchak CFE II",-1622,8)
  {
   z=105;
   Outlier(0.05);
  };
  C_Date("Ruppert",-515,335)
  {
   z=74;
   Outlier(0.05);
  };
  C_Date("WRAn",325,65)
  {
   z=48;
   Outlier(0.05);
  };
  C_Date("Opala",615,40)
  {
   z=37;
   Outlier(0.05);
  };
  R_Date("11-13",785,45)
  {
   z=12;
   Outlier(0.05);
  };
  Combine()
  {
   R_Date("2.6- 4.6",170,30);
   C_Date("3-3.6",1870,7);
   z=3.6;
   Outlier(0.05);
  };
  Curve("Bomb21NH1","Bomb21NH1.14c");
  C_Date("2.25-3",1901,4.)
  {
   z=3;
   Outlier(0.05);
  };
  C_Date("1.5-2.25",1930,2)
  {
   z=2.25;
   Outlier(0.05);
  };
  C_Date("1-1.5",1946,2)
  {
   z=1.5;
   Outlier(0.05);
  };
  C_Date("0.5-1",1965,1.5)
  {
   z=1;
   Outlier(0.05);
  };
  C_Date("0-0.5",1990,1)
  {
   z=0.5;
   Outlier(0.05);
  };
  C_Date("Surface",2013,0.5)
  {
   z=0;
   Outlier(0.05);
  };
  Boundary();
 };
};
```

Figure S1: Cascade Lake core images, lithological units, wet bulk density (WBD), organic matter (OM) and biogenic silica (BSi) data from Steen (2016).

.

Figure S2: Optical microscopy glass shard images for the five unimodal cryptotephra populations reported from Cascade Lake.

a) CL 37 - Opala

b) CL 48 - WRAn

Figure S2(cont.): Optical microscopy glass shard images for the five unimodal cryptotephra populations reported from Cascade Lake.

c) CL 74 - Ruppert

d) CL 96 - Unknown

e) CL 105 - Aniakchak CFE II

Figure S3: Bayesian Tau_Boundary probability density function plots derived using OxCal v4.4 and IntCal20 for the age of Aniakchak CFE II tephra with: all ^{14}C dates are included (grey right-hand distribution); two ^{14}C dates removed (green central distribution); and all but two ^{14}C dates and the NGRIP ice core chronology age (Pearce et al., 2017) (blue left-hand bar). See Table S5 for the ages used for this model.

Figure S4: Bayesian Tau_Boundary probability density function plots derived using OxCal v4.4 and IntCal20 for the age of OP tephra, Opala, Kamchatka. See Table S5 for the ages used for this model.

Figure S5: Major element glass geochemical biplots showing wt.% SiO_2 vs K_2O and FeO vs CaO for samples with multiple geochemical populations. (**a**) CL-0 and CL-2; (**b**) CL-4 and CL-7; (**c**) CL-31 and CL-61; (**d**) CL-0, -2, -4 and -7 plotted both by sample and by geochemical correlation with a source volcano or region.

Figure S6: OxCal age-depth output for the final Bayesian model for Cascade Lake. The students'-t outlier analysis results shown good agreement.

Posterior = likelihood of being an outlier
(original assumption is 5, or 1 in 20 chance)

Element	Ok	Outlier	Prior	Posterior	Model	Type
348.5 – 351			5	6	General	t
303-304			5	9	General	t
233.5-235.5			5	4	General	t
197-199			5	4	General	t
138-140			5	4	General	t
Aniakchak CFE II			5	3	General	t
Ruppert			5	4	General	t
WRAn			5	4	General	t
Opala			5	3	General	t
11-13			5	4	General	t
3-3.6			5	3	General	t
2.25-3			5	3	General	t
1.5-2.25			5	4	General	t
1-1.5			5	4	General	t
0.5-1			5	3	General	t
0-0.5			5	2	General	t
Surface			5	2	General	t