

1 **S1 Pre-requisite:**

2 **S1.1 Supported Platforms:**

- 3 • Windows: Windows 10/11 (64-bit) — recommended
- 4 • macOS: 64-bit macOS supported by your MATLAB release

5 **S1.2 Software requirements:**

6 The CoRSEER was developed using MATLAB App Designer 2025a. We recommend that the user install the provided toolbox before

7 installing CoRSEER and install a version of MATLAB that is the same or newer version of MATLAB than 2025a. If not installed, it

8 might lead to unexpected errors during execution.

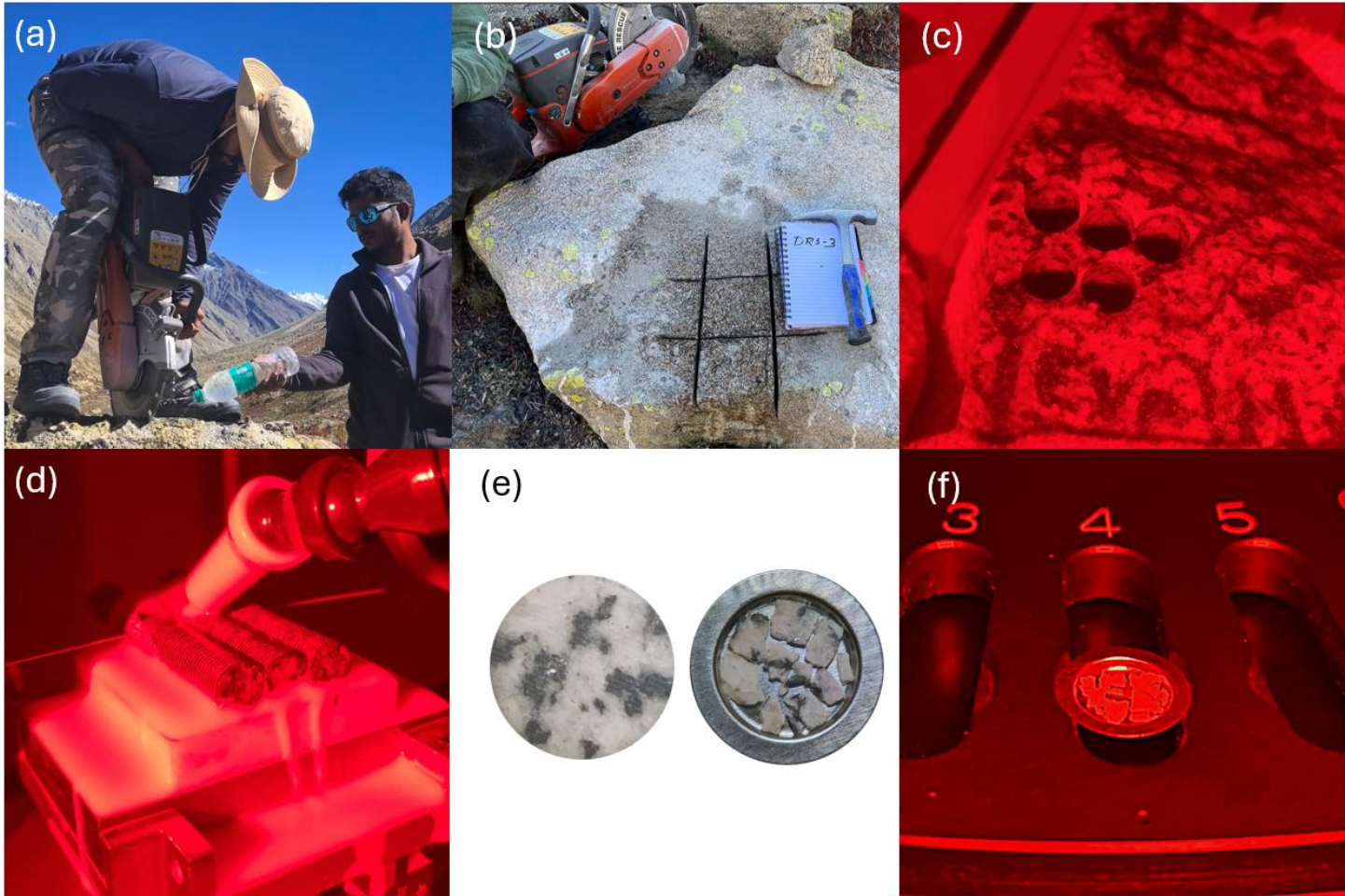
- 9 1. Statistical and Machine Learning Toolbox
- 10 2. Curve Fitting toolbox
- 11 3. Parallel computing toolbox
- 12 4. Optimisation toolbox

13 Data I/O: Microsoft Excel should be installed on the computer for editing and accessing input files.

14 **S1.3 Hardware requirements:**

15 Table S1: System requirements for the appropriate execution of CoRSEER.

Category	Minimum (basic use)	Recommended (research workflow)	High-Performance (large batches)
CPU	2 cores, ≥ 2.5 GHz	4–8 cores, ≥ 3.0 GHz	> 16 cores for parallel computing
RAM	8 GB	16 GB	32 GB
Storage	25 GB free	SSD, 100 GB free	NVMe SSD, ≥ 100 GB free
Display	1366×768	1920×1080	1920×1080
GPU	Not required	Not required	Not required



18

19 Figure S1: a-b) Drilling of a 10×10×10 cm rock block from the boulder under water lubrication. c) Coring of the rock block perpendicular
20 to the surface up to 4-5 cm and a diameter of 10 mm. d) Slicing of the core using a diamond wire-saw in oil lubrication conditions by
21 preserving the depth information. e) Placing the rock slice on a metal stainless steel cup by breaking it into smaller fragments. f)
22 Placement of the metal cup on the carousel for luminescence measurement.

23 Select a sampling location that is not shaded by any object and not heavily shielded by surrounding topography, boulders, vegetation,
24 or structures during any significant part of the day, so that the surface remains well exposed to sunlight (Fig. S1a). Prefer a sampling
25 surface that is flat and horizontal. Prefer sampling from a clean, well-exposed surface and avoid locations with intense weathering that
26 produce loose material. Avoid sampling where sand, granular weathering products, or other debris cover the host rock surface, as this
27 cover alters adequate light exposure and can disturb the near-surface luminescence signal. Avoid sampling from surfaces with lichen
28 cover or any natural coating layer and record the presence or absence of lichen or other coverings if complete avoidance is not possible
29 (Fig. S1b). Follow the site-selection logic used in TCN rock exposure dating, but implement the key contrast that RSELd requires:
30 collecting a rock block rather than chipping only the exposed surface. Plan sampling to be well away from edges to minimise lateral
31 light penetration into the rock, which can distort the luminescence depth profile, and strictly avoid coring or cutting from double-exposed
32 surfaces, such as corners, thin slabs, or blocks with two free faces, where exposure can occur from more than one direction (Fig. S1b).

33 Before sampling, measure and record the strike and dip at the sampling point on the rock surface, because surface orientation controls
34 incident sunlight and may require correction during calibration or comparison. Prefer sampling from locally flattened portions of the
35 surface and avoid undulating microtopography to improve coring or cutting precision and reduce microshading effects. Choose a cutting

approach that preserves the exposed surface as completely undisturbed, as it serves as the reference boundary for the depth profile. If a machine cutter is used, perform cutting with continuous water lubrication to minimise frictional heating between the blade and rock, thereby reducing thermally induced signal loss; maintain consistent water flow throughout cutting; and avoid dry cutting at any stage. Immediately after extraction, protect the surface portion from chipping, spalling, abrasion, or any mechanical damage, because removal of even a small thickness can truncate the near-surface part of the luminescence profile. Prefer a block size of approximately $10 \times 10 \times 10$ cm, then immediately wrap the extracted block in aluminium foil and place it into a light-protected bag or container, keeping it light-protected during transport and storage to avoid signal loss. Where feasible, create a calibration surface on a nearby portion of the same rock body by removing approximately 5 cm of exposed surface material to generate a fresh surface for known-duration exposure, ensure that the calibration and unknown surfaces match in orientation as closely as possible (including strike and dip), and prefer resampling of the calibration surface after a minimum of one year to integrate a complete annual cycle of insolation exposure and reduce seasonal bias in calibration.

S3 Sample Preparation:

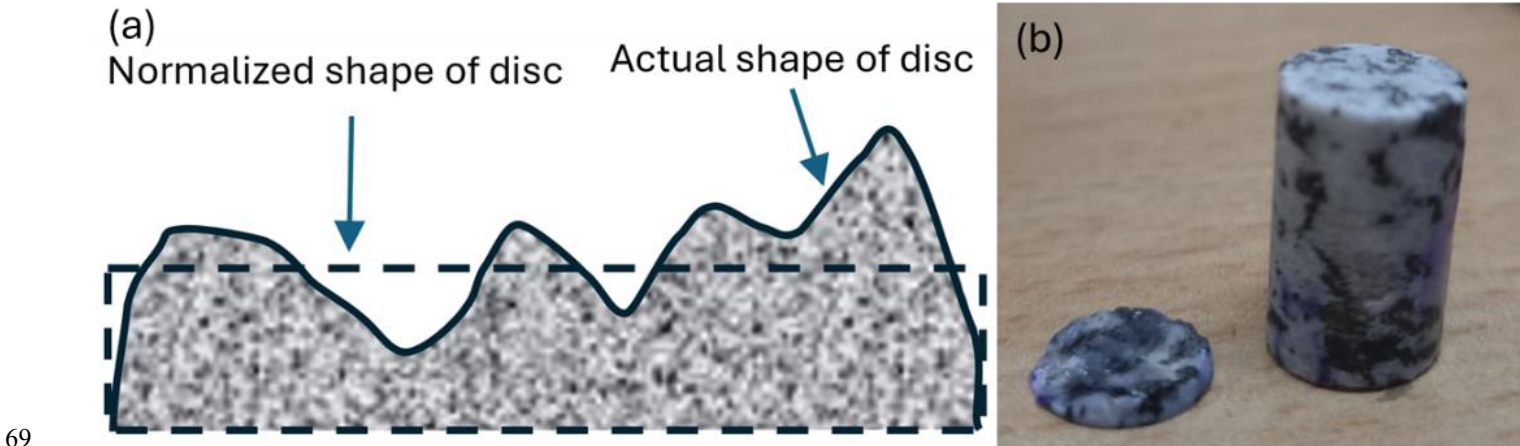
S3.1 Coring Strategy:

Perform coring under subdued red-light conditions and with continuous water lubrication to minimise any inadvertent optical stimulation during sample preparation (Fig. S1c). Core perpendicular to the flat rock surface, and before drilling, flatten and level the surface using a bubble level so that the drilling axis remains truly normal to the surface. Use a core diameter of 1 cm and drill to a minimum depth of 40–50 cm, ensuring the coring direction remains perpendicular to the surface throughout. Preferentially extract at least two cores from each rock block to provide replication, and place the coring points as close to the block centre as feasible; if central coring is not possible, position cores at least 1–2 cm away from edges, and increase this offset to at least 4–5 cm if the edge may have been laterally exposed in the past. Immediately after extraction, mark each core unambiguously for top and bottom orientation to prevent confusion during subsequent slicing, then dry the cores and store them in light-protected bags until further processing.

S3.2 Slicing strategy:

Conduct slicing under subdued red-light conditions to minimise any inadvertent optical stimulation. Slice each core perpendicular to the core axis at nominal 1 mm intervals; if the rock is brittle, increase the interval (e.g., 2–3 mm) to avoid fragmentation while keeping the interval constant within a core whenever possible (Fig. S1d). Immediately after each cut, assign an unambiguous identifier to every slice (core ID + slice number counted from the exposed surface inward) and maintain the top–bottom orientation mark carried from the core, so that no interchange can occur during handling, storage, or measurement. Measure the thickness of each slice and define the depth of that slice using a centre-to-centre convention, i.e., assign the representative depth of slice i as the cumulative thickness of all overlying slices and material depth loss during cut, plus half of the slice thickness. Based on the existing LDPs from literature, we suggest slicing up to a depth of 30cm, but the depth might change based on exposure conditions. The surface slice (slice 1) is a special case because surface undulation biases Vernier calliper measurements toward the maximum thickness; thus, we present a bulk density method for top slice thickness estimation.

68 **S3.2.1 Top slice measurement using bulk density:**



70 Figure S2: (a) The cross-section of the top disc due to roughness and undulation, which can lead to an error in depth measurement using
71 Vernier callipers. We suggest using the disc's normalised shape. (b) The top slice and cylindrical core are used for density measurement.

72

73 There are a few assumptions we propose to perform the top slice measurement using bulk density: 1) The density of rock is constant
74 throughout. 2) 5% of the weight loss of the top slice in the coring operation. 3) The non-uniform shape of the top disc is normalised to
75 an ideal disc as shown in Fig. S2a. To calculate the average thickness of the surface slice, first determine the density of the core, which
76 was cut from the main core using an oven-dried standardised subsample. Cut a representative piece of the core to a standard geometry
77 (suggested: approximately 2 cm height and 1 cm diameter as shown in Fig S2b), oven-dry both the subsample and the slices for at least
78 24 hours at approximately 50 °C, and measure the oven-dried weight on a high-precision balance. Calculate the core density using
79 $\rho_{core} = \frac{4W_{core}}{\pi d_{core}^2 h_{core}}$, where W_{core} is the dry weight of the standardised core segment, d_{core} is its diameter, and h_{core} is its height. Next,
80 measure the oven-dried weight of the top slice on a high-precision balance and apply a correction for material loss during coring by
81 assuming 5% mass loss; incorporate this correction by using a factor of 0.95 in the thickness calculation. Estimate the average slice
82 thickness using $h_{slice_avg} = \frac{4W_{slice}}{0.95 \rho_{core} \pi d_{core}^2}$, where W_{slice} is the dry weight of the slice and d_{core} is the core diameter. For improved
83 precision, it is advisable to cut the top slice slightly thicker than the nominal interval so that the mass-based thickness estimate remains
84 robust against small-scale surface undulations and minor edge losses.

85 **S3.3 Luminescence Measurement:**

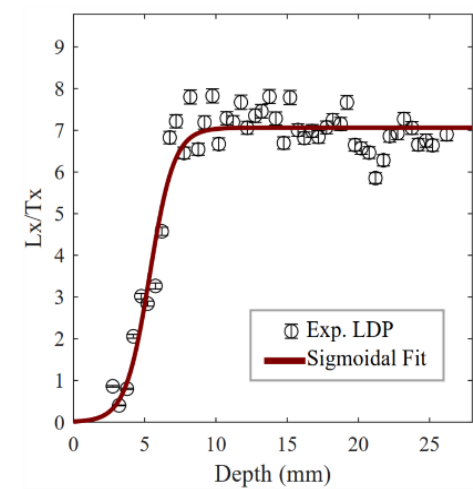
86 For luminescence measurements, rock slices are gently broken into small chips and arranged flat in stainless-steel cups to ensure
87 excellent thermal contact and stable heating, yielding more reliable luminescence signals (Elkadi et al., 2021). Luminescence is then
88 measured using the modified rock-slice SAR protocol (Jenkins et al., 2018) (Table S2), optimised for high-quality RSEL depth profiles.
89 The key distinctions from standard feldspar SAR are a long pause time (~100 s) before measurement, longer signal integration, and a
90 reduced heating rate. The example shown is a luminescence depth profile with depth on the x-axis and L_x/T_x on the y-axis, where the
91 maroon curve represents a sigmoidal fit to the experimental data.

92 Table S2: The modified SAR protocol for luminescence measurements of rock slices.

Step	Description	Signal
1	No dose in cycle 1 – different regenerative doses in later cycles	

2	Preheat to 250 °C at 1 °C/s and hold for 100 s	
3	Heat to 50 °C at 1 °C/s, pause for 100 s, and measure IRSL for 200 s	L_{x50}
4	Heat to 225 °C at 1 °C/s, pause for 100 s, and measure IRSL for 200 s	L_{x225}
5	Test dose (34 Gy)	
6	Preheat to 250 °C at 1 °C/s and hold for 100 s	
7	Heat to 50 °C at 1 °C/s, pause for 100 s, and measure IRSL for 200 s	T_{x50}
8	Heat to 225 °C at 1 °C/s, pause for 100 s, and measure IRSL for 200 s	T_{x225}
9	Heat to 280 °C at 1 °C/s, pause for 100 s, and measure IRSL for 200 s	
10	Repeat cycles 1–9 for different regenerative doses in step 1	

93



94 Figure S3: The experimental LDP (hollow circle) of 1 a 1-year-old sample established using modified SAR. The datapoints show
95 insignificant scatter and smoothly follow the sigmoidal curve.

96 **S4 Modelling aspect and CoRSEER.**

97 In Rock surface exposure luminescence dating, forward-inverse modelling is used to estimate calibration parameters, past exposure
98 duration and erosion rate by simulating how luminescence signals evolve within the rock over time and comparing them to measured
99 depth profiles. The forward model simulates signal accumulation and bleaching using known or experimentally derived parameters. The
100 inverse model adjusts these parameters iteratively to minimise the misfit and is quantified via the chi-square method between observed
101 and modelled luminescence-depth profiles. The sigmoidal curve fitting (SCF) is executed in all three parts of CoRSEER before initiation
102 of the inversion procedure, and the experimentally derived LDP is normalised by a factor. The sigmoidal curve-fitting strategy is
103 thoroughly discussed in Section 3.1 of the manuscript.

104

S4.1 CoRSEER-I: Calibration

The first operation of CoRSEER is to evaluate kinetic parameters, $\overline{\sigma\phi}_o$ and μ using a known age sample. This operation follows the single calibration method. At first, we establish the sigmoidal fitting parameters corresponding to the calibration LDP. The b value then normalises the whole calibration LDP, and the plateau point is detected. Further, we perform the inversion (Pathan et al., 2024) to evaluate kinetic parameters. The GUI of CoRSEER-I is shown in Fig. S4.

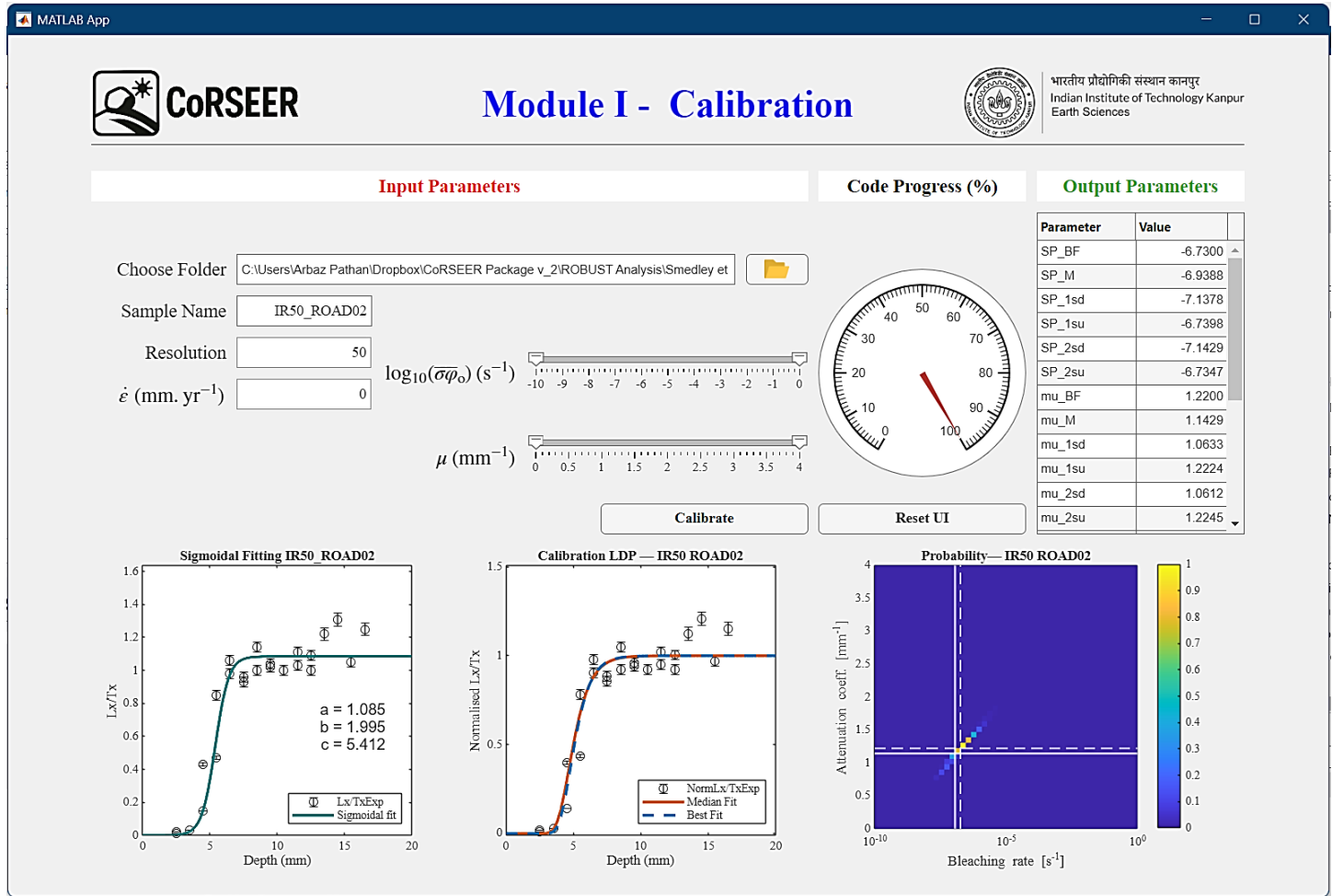


Figure S4: The GUI of CoRSEER-I used for calibration of kinetic parameters. The GUI includes multiple input bars in its left section, the progress bar and output parameters in the right, and the diagnostic plots at the bottom.

S4.1.1 Input Parameters:

In CoRSEER Part I, the key input parameters for forward modelling are the bleaching rate ($\overline{\sigma\phi}_o$) and the attenuation coefficient (μ). Based on observational constraints from previous studies and empirical datasets, $\overline{\sigma\phi}_o$ is defined over a range of 10^{-10} to 10^0 s^{-1} for calibration. The attenuation coefficient μ , which governs light penetration into the rock surface, is set as a customisable input from 0 to 4, with an upper limit of 4 justified by empirical saturation observed in Smedley's IR50_ROAD01 sample (Table 3). These parameter bounds are user-editable within the CoRSEER interface, allowing flexibility while retaining physical realism grounded in field-tested data.

S4.1.1.1 Input Excel File

The CoRSEER Part I module requires an Excel file structured with specific luminescence and depth-related parameters for accurate execution and model calibration. The prescribed format of the Excel file is given below:

Column 1: contains unique, non-zero depth values (in mm); duplicate or identical depths will trigger interpolation errors during model execution. If duplicate depths are unavoidable, we recommend offsetting one by a minimal value (e.g., 10^{-5} mm), which will not affect the modelling outcome but ensure algorithmic stability.

Column 2: contains Lx/Tx values, all strictly non-zero and non-negative. To prevent failures in the Monte Carlo-based sigmoidal fitting, any true zero or negative value should be replaced with a small placeholder (e.g., 10^{-6}).

Column 3: Contains Lx/Tx error, which is also required to be non-zero for the same reason.

Column 4: This specifies the order of kinetics (r) for the sample, typically determined from protocol-specific measurements.

Column 5: The dose rate (Gy/a), derived from ICP-MS or high-resolution gamma spectrometry and final dose rate should be established using protocol mentioned in Sohbati et al. 2018 (Sohbati et al., 2018).

Column 6: The characteristic dose (Gy), both of which can be derived from laboratory techniques such as Dose Response Curve (DRC), Murray and Wintle 2006.

Column 7: Known calibration age (in years), which directly influences execution time, as the forward simulation duration scales proportionally with age.

Adhering to this structure ensures smooth integration with CoRSEER's computation engine and prevents common runtime errors.

S4.1.1.2 Matlab GUI options

The CoRSEER Part I GUI provides a user-friendly interface with key input options to streamline calibration.

Input Box 1: The folder selection option lets users change the working directory from MATLAB's default to the folder containing the experimental data. The calibration file must be named OSL_Calib.xlsx, and the sample name will be in the sheet name of the Excel file. For example, here we have named the samples as XX_SAMPLENAME, where XX = IR50, pIRIR150, or pIRIR225, identifying the signal type.

Input Box 2: The second input is m , which determines the grid density for calibration by dividing the ranges of $\overline{\sigma\varphi}_o$ and μ into $m \times m$ pairs for surface fitting. A typical value of $m = 1000$ balances accuracy and computational efficiency.

Input Box 3: Third, the Sample Name field enables dynamic matching to the corresponding sheet within OSL_calib.xlsx, making the system modular and scalable for multiple datasets.

Input Box 4: Lastly, the erosion rate input adds flexibility by allowing the user to define mean surface erosion during calibration. While freshly exposed or artificially prepared calibration samples typically assume zero erosion, older samples or those from highly eroding landscapes may require non-zero values, which is an essential factor often overlooked in earlier models, and this is thoroughly discussed in Section 5 of the Manuscript. By incorporating this parameter, CoRSEER-I improves physical realism and enhances parameter inversion precision.

S4.1.2 Mechanism

CoRSEER Part I begins by fitting the experimental luminescence-depth profile using a sigmoidal function to extract the parameter a , which represents the LDP plateau magnitude. This value is used to normalise the Lx/Tx data across the profile. We also establish the σ of the absolute of residual values by subtracting the SCF curve and experimental dataset. The δ is later used as a noise threshold in inversion routines.

S4.1.2.1 Forward modelling:

$$\frac{d\bar{n}}{dt} = \left\{ \frac{\dot{D}}{D_o} (1 - \bar{n}) \right\}_{growth} - \left\{ \bar{n}^r \overline{\sigma\varphi}_o e^{-\mu x} \right\}_{decay} + \left\{ \varepsilon \frac{d\bar{n}}{dx} \right\}_{erosion} \quad (S1)$$

The above ODE is solved using the explicit Euler Forward time scheme (a first-order explicit numerical method. The time is discretised into small steps $\Delta t = 0.01 \text{ yr}$, and depth is discretised into a fixed grid (e.g., 75 depth points from the surface down to a depth slightly beyond the deepest measurement). The initial condition at $t = 0$ assumes the rock was fully saturated with luminescence at all depths ($\bar{n} = 1$) which is derived from the assumption that there is no prior light exposure before the current exposure period. At each time step, the normalised luminescence signal (NLS) at each depth is updated as:

$$\bar{n}_{new} = \bar{n}_{old} + \left[\frac{\dot{D}}{D_o} (1 - \bar{n}) - \bar{n}^r \overline{\sigma\varphi}_o e^{-\mu x} + \varepsilon \frac{d\bar{n}}{dx} \right] \times \Delta t \quad (S2)$$

The advection term is only included if the $\varepsilon > 0 \text{ mm/a}$. After each step, any numerical overshoot is corrected by constraining the solution. $\bar{n}$ within $[0,1]$ (the values outside this range are set to 0). This ensures that simulated NLS remains physically valid. The iteration is performed up to the t of the calibration sample for each pair ($m \times m$) of incorporated. $\overline{\sigma\varphi}_0$ and μ .

S4.1.2.2 Inverse Modelling

To calibrate the unknown parameters, the synthetic profile is compared to the experimental profile. For each pair of parameters $\overline{\sigma\varphi}_0$ and μ on a defined grid (spanning the user-specified range in $\log_{10}(\overline{\sigma\varphi}_0)$ and linear μ with resolution of m in each dimension, a synthetic NLS profile is generated as described above. The model then computes a misfit between the synthetic ($\bar{n}_{sim}$) and experimental ($\bar{n}_{obs}$) LDP. Furthermore, every computed misfit is then converted into a likelihood score assuming a Gaussian error model as:

$$\chi^2 = \sum_{i=1}^l \frac{((\bar{n})_{obs}^{(i)} - (\bar{n})_{sim}^{(i)})^2}{\sigma^2} \quad (S3)$$

$$likelihood = P = e^{-\chi^2/2} \quad (S4)$$

$$P_{norm}^k = \frac{P_k}{P_{max}} \quad (S5)$$

where l is the number of experimental data points, δ is the standard deviation of the residual of SCF and experimental LDP, P_k is the likelihood value associated with the synthetic LDP numbered as k , P_{max} is maximum-likelihood, which is associated with the best-

fitting synthetic LDP and P_{norm}^k is the normalised likelihood value associated with the synthetic LDP numbered as k . The best paths were selected by applying the Monte-Carlo acceptance criterion as $P_{norm}^k > rand(m,m)$. Finally, the CoRSEER-I establishes the values of best-fit, median, $\pm 1\delta$, and $\pm 2\delta$ for both $\log_{10}(\overline{\sigma\varphi_0})$ and μ using confidence intervals. It must be noted that both the best fitted value and median of accepted values after the Bayesian approach should, ideally, overlap with each other. However, if the matrix resolution is low or the input range is too broad or the input parameter range is biased toward higher or lower values, the median may deviate substantially from the best-fit value. Thus, for precise and accurate output, the input calibration parameter range and the number of pairs must be adjusted in such a way that the best-fit and median should be closest to each other.

S4.1.3 Output

CoRSEER Part I generates three output categories after calibration.

Output 1: A dedicated Excel file (var_cal_[SampleName].xlsx) is saved, containing all key results from sigmoidal fitting and calibration parameter estimation. This serves as the primary record of the calibration outcome.

Output 2: All internal MATLAB variables used during computation are saved in a .mat file, enabling complete reproducibility and post-processing flexibility.

Output 3: The software exports four diagnostic plots: (1) a sigmoidal fit of the experimental Lx/Tx vs depth profile, with Lx/Tx error bar, (2) a plot comparing experimental data with modelled profiles for both best-fitting and median values of calibration parameters, and (3) a surface plot depicting the likelihood space of $\overline{\sigma\varphi_0}$ and μ , where the colour bar represents normalised chi-square misfit values. These visual outputs provide an intuitive and quantitative assessment of the calibration quality and fitting precision.

S4.2 CoRSEER-II: Apparent age estimation

CoRSEER-II is designed to estimate the exposure age of a rock surface from its luminescence-depth profile (LDP) using kinetic parameters calibrated in CoRSEER-I, along with experimental inputs such as dose rate, order of kinetics, and characteristic dose. The forward model systematically simulates synthetic luminescence profiles over a defined age range, and each is fitted to the normalised experimental LDP of unknown age samples. The age corresponding to the minimum chi-square misfit is identified as the best-fitting exposure age.

In addition to estimating exposure age, CoRSEER also determines the maximum datable age, the upper temporal bound beyond which the LDP cannot propagate further into rock and reaches equilibrium. This is achieved through a kinetic tracking of the depth of half-saturation $x_{0.5}$ over time. By simulating its propagation as exposure duration increases, the model identifies the maximum depth of luminescence penetration. The exposure time corresponding to $x_{ss} - 1/\mu$ value is defined as the maximum datable age. This approach ensures a physically grounded limit to the model's temporal resolution. The GUI of CoRSEER-II is shown in Fig. S5.

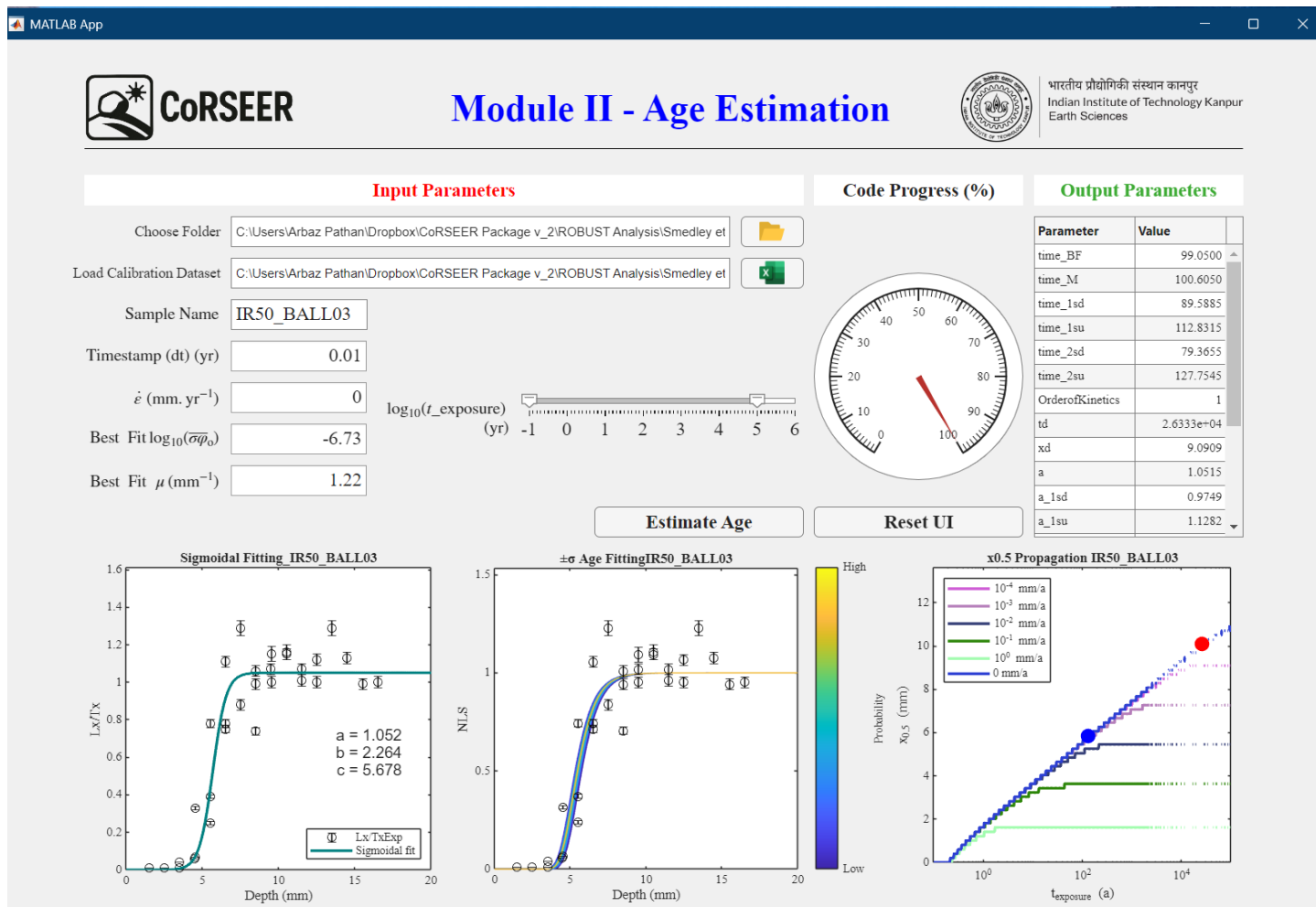


Figure S5: The GUI of CoRSEER-II used for estimation of apparent age and maximum dateable age. The GUI includes multiple input bars in its left section, the progress bar and output parameters in the right, and the diagnostic plots at the bottom.

S4.2.1 Input Parameters

S4.2.1.1 Excel Format

The input Excel file for CoRSEER Part II must adhere to the same structure defined in Section 1.1.1.1 of CoRSEER-I. We must note that for both the calibration sample and the unknown-age sample, the kinetic order should be equivalent. The sheet must contain non-zero, unique depth values along with corresponding Lx/Tx and error columns. Duplicate depths should be corrected by a minimal offset (e.g., 10⁻⁵ mm) to avoid interpolation failure during modelling. Crucially, the age column should be left blank, as the goal of this module is to estimate exposure age. The file must be named OSL_unknown.xlsx, and each sheet within it should carry the exact name of the sample being analysed. We must note that for both the calibration sample and the unknown-age sample, the kinetic order should be equivalent.

This sheet name is used by the software to extract and process the corresponding luminescence-depth profile.

S4.2.1.2 GUI Options

In the MATLAB App, the user interface allows flexible control over the modelling process.

Input Box 1: Load the calibration dataset (var_cal_[SampleName].xlsx) generated from CoRSEER Part I. Upon loading, the best-fit kinetic parameters $\overline{\sigma\varphi}_o$ and μ are automatically read and displayed in their respective GUI fields.

Input Box 2: The next step is to direct the software to the folder containing the OSL_unknown.xlsx file. Once the correct folder is set, the user must enter the sample name in the designated GUI field; this name identifies the sheet in the Excel file containing the unknown LDP.

Input Box 3: This is a range bar which includes a 'Log10' field that defines the logarithmic age search range (e.g., 10^{-1} to 10^6 years), which governs how the forward model simulates profiles. Since simulation time scales with this range, a higher upper bound will result in longer computations.

Input Box 4: The 'timestamp' or Δt value represents the time step for simulation and should ideally be 100 times smaller than the minimum age (*i.e.*, $dt = t_{min} / 100$) to ensure sufficient temporal resolution.

Input Box 5: Another field allows the user to enter an erosion rate; if this value is unknown, it should simply be set to 0, indicating no erosion is assumed.

S4.2.2 Mechanism

S4.2.2.1 Forward modelling:

$$\frac{d\bar{n}}{dt} = \left\{ \frac{\dot{D}}{D_o} (1 - \bar{n}) \right\}_{growth} - \left\{ \bar{n}^r \overline{\sigma\varphi}_o e^{-\mu x} \right\}_{decay} + \left\{ \varepsilon \frac{d\bar{n}}{dx} \right\}_{erosion} \quad (S6)$$

The forward modelling in the current module uses the Euler forward time scheme, which we discussed thoroughly in earlier sections. The main difference is that in CoRSEER-I, we solve the above equation up to known calibration t at time stamp $\Delta t = 0.01 \text{ yr}$ for each pair ($m \times m$) of $\overline{\sigma\varphi}_o$ and μ . Furthermore, we identify a misfit for each pair and search for pairs that yield the best-fitting LDP. In CoRSEER-II, we solve the equation by fixing calibration parameters as the best-fitting pairs of $\overline{\sigma\varphi}_o$ and μ for each unknown t within the provided exposure age range for multiple erosion rate scenarios as discussed in Section 3.2.3. We track $x_{0.5}$ as we solve the equation and plot it for different erosion scenarios.

S4.2.2.2 Inverse Modelling

To calibrate the unknown parameters, the synthetic profile is compared to the experimental profile. For each t (spanning the user-specified range in log-space), a synthetic NLS profile is generated. The model then computes a misfit between the synthetic ($\bar{n}_{sim}$) and experimental ($\bar{n}_{obs}$) LDP. Furthermore, every computed misfit is then converted into a likelihood score assuming a Gaussian error model as:

$$\chi^2 = \sum_{i=1}^l \frac{((\bar{n})_{obs}^{(i)} - (\bar{n})_{sim}^{(i)})^2}{\sigma^2} \quad (S7)$$

$$likelihood = P = e^{-\chi^2/2} \quad (S8)$$

$$P_{norm}^k = \frac{P_k}{P_{max}} \quad (S9)$$

where l is the number of experimental data points, δ is the standard deviation of the residual of SCF and experimental LDP, P_k is the likelihood value associated with the synthetic LDP numbered as k , P_{max} is maximum-likelihood, which is associated with the best-fitting synthetic LDP and P_{norm}^k is the normalised likelihood value associated with the synthetic LDP numbered as k . The best paths were selected by applying the Monte-Carlo acceptance criterion as $P_{norm}^k > rand(m, l)$. Finally, the CoRSEER-II establishes the values of best-fit, median, $\pm 1\delta$, and $\pm 2\delta$ for t using confidence intervals.

S4.2.3 Output

CoRSEER-II generates three primary types of output: important variable results, output MATLAB data files and diagnostic plots.

Output 1: A dedicated Excel file (var_age_[SampleName].xlsx) is saved, containing all key results from sigmoidal fitting and apparent exposure age, maximum dateable age and many other variables, and serves as the primary record of the outcome.

Output 2: The data outputs include a .mat file containing all key modelling variables

Output 3: The three diagnostic plots that visually capture different aspects of the modelling quality and outcomes. 1) the SCF to the raw Lx/Tx wrt depth data. 2) Presents the evolution of $x_{0.5}$, the depth of half-saturation as a function of time, demonstrating how it asymptotically approaches steady state. This time-dependent propagation is plotted for different erosion scenarios. 3) The uncertainty envelope visualisation displays the suite of synthetic LDP that meet the selection criterion ($\chi^2_{norm} > rand(m, 1)$), overlaid on the experimental LDP. These profiles are colour-coded by normalised chi-square value, illustrating the spread of acceptable fits from -1δ to $+1\delta$ and providing a detailed visual sense of fitting confidence.

S4.3 CoRSEER-III: Erosion estimation

CoRSEER Part III is designed to estimate erosion histories of rock surfaces by inverting luminescence-depth profiles using calibrated kinetic parameters and experimental data. It explores combinations of erosion rates and erosion onset times to determine how different erosion scenarios affect luminescence signal evolution. By fitting synthetic depth profiles generated under these scenarios to experimental data, CoRSEER identifies the most plausible erosion histories and quantifies uncertainties. The GUI of CoRSEER-III is shown in Fig. S6.

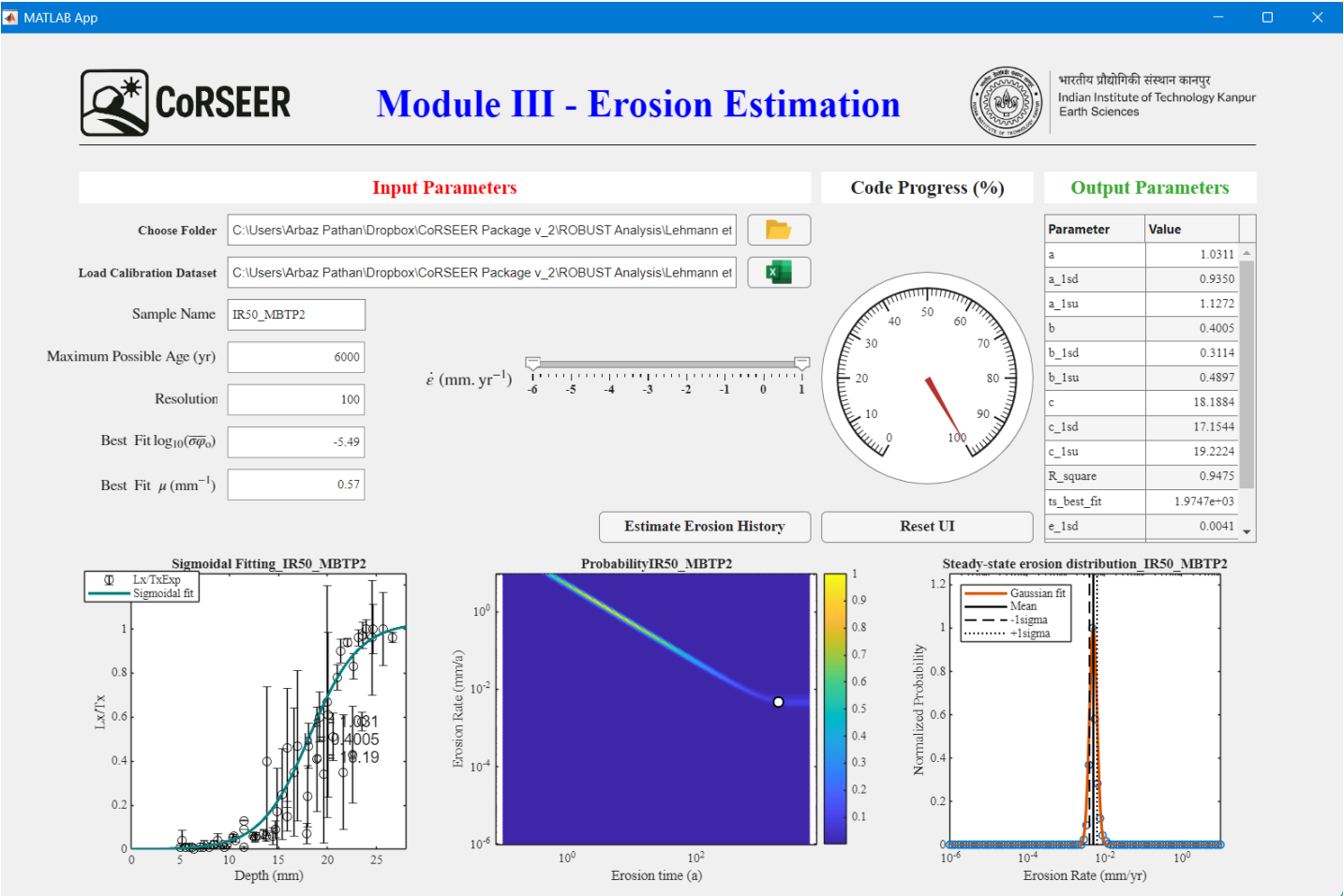


Figure S6: The GUI of CoRSEER-III used for estimation of steady state erosion rate and associated exposure age. The GUI includes multiple input bars in its left section, the progress bar and output parameters in the right, and the diagnostic plots at the bottom.

S4.3.1 Input Parameters

S4.3.1.1 Excel File

The input Excel file format for Part III follows the same structure as defined in CoRSEER Part II (Age Estimation). It must contain non-zero, non-repeating depth values, normalised luminescence signals, associated errors, and other experimental constants. The age cell should remain blank, and the sheet name should match the sample name to be processed.

S4.3.1.2 GUI Input

The graphical user interface (GUI) for CoRSEER Part III provides several user-defined fields to control the erosion modelling process.

Input box 1: Allows loading of the calibration dataset (var_cal_[SampleName].xlsx), from which the best-fit kinetic parameters $\overline{\sigma\phi}$ and μ are automatically retrieved and displayed, though users may also input them manually.

Input box 2: The second field sets the working directory containing the target OSL_unknown.xlsx file. The sample name entered must exactly match the sheet name within this Excel file to ensure correct data loading.

Input box 3: Defines the upper bound of exposure duration over which erosion scenarios are evaluated.

Input box 4: The parameter m controls the resolution of the erosion-onset grid: larger m values result in finer sampling in logarithmic space.

Input box 5 (range): Users also define the logarithmic search range for the erosion rate, typically between 10^{-6} and 10^1 mm/a, consistent with geologic-scale erosion rates.

Together, these inputs allow the user to construct a high-resolution, physically constrained erosion–exposure landscape tailored to the calibration and luminescence characteristics of each sample.

S4.3.2 Mechanism:

CoRSEER-III is designed to estimate erosion histories of rock surfaces by inverting luminescence-depth profiles using calibrated kinetic parameters and experimental data. It explores combinations of erosion rates and erosion onset times to determine how different erosion scenarios affect luminescence signal evolution. By fitting synthetic depth profiles generated under these scenarios to experimental data, CoRSEER identifies the most plausible erosion histories and quantifies uncertainties.

Using logarithmic spacing defined by the user-provided m value, CoRSEER creates two vectors: erosion rates ($\dot{\epsilon}$) and exposure durations t . For each ($\dot{\epsilon}$, t) pair, it simulates luminescence-depth profiles using a two-phase kinetic model: Phase 1 with no erosion (from 0 to purely exposure period (t_o)), and Phase 2 with constant erosion, $\dot{\epsilon}$ (for the period of t_s) ($t = t_o + t_s$, total exposure age). The forward model integrates signal growth, bleaching, and advective loss due to erosion using a finite difference method. Inverse modelling establishes the misfit between modelled and experimental profiles, which is evaluated using chi-square statistics, as described in earlier sections. The chi-square matrix is then converted into a likelihood field, which is normalised to produce P_{norm}^k . Posterior sampling is performed by filtering. $P_{norm}^k > \text{rand}(m, m)$, generating distributions for erosion rate and onset time.

The main objective was to extract the minimum erosion-onset time at which the best-fit erosion solution becomes stable, and then estimate the representative erosion rate with uncertainty using a Gaussian probability distribution. The analysis was performed using the final misfit/probability matrix generated during the erosion-age modelling step. At first, we search for the best-fitting values in each column associated with each t_s . The unique best-fitting pairs of $\dot{\epsilon}$ and t_s is smoothed using the moving median interpolation method. The algorithm searches for the point where the slope of this line becomes zero, and then the closest matrix associated with that point is chosen for the Gaussian distribution operation to calculate the mean and $\pm 1\delta$. It must be noted that the in-built parameter to detect the onset time of erosion is adjusted based on the analysed dataset and might fail in some exceptional circumstances. In that case, we suggest using a matrix of the erosion rates, the onset time of erosion and the resulting $norm_chi$ can be used to perform the Gaussian probability distribution operation manually. We recommend the user to adjust the limits of erosion and/or increase resolution to enhance the quality of the dataset.

S4.3.3 Output

CoRSEER Part III produces comprehensive results in three formats.

Output 1: The Excel output sheet (var_Er_[SampleName].xlsx) contains all key results from sigmoidal fitting.

Output 2: A .mat file is generated containing all relevant variables, including all the resulting variables.

Output 3: The three diagnostic plots: 1) SCF plot, which shows the fitting of (L_x/T_x) vs depth. 2) Erosion Surface Plot – A 3D surface plot with erosion rate on the Y-axis, erosion onset time on the X-axis, and normalised chi-square as the Z-axis. We must mention that the results must be picked manually from the plot, and the solution range is very broad (for the same LDP, both transient and steady-state solutions are possible). (3) Gaussian distribution of the erosion rate column associated with the minimum onset time of erosion.

These outputs collectively provide a full erosion-exposure landscape, allowing users to visualise and interpret plausible erosional histories constrained by luminescence data.

S5 Covariance of $\overline{\sigma\phi}_0$ and μ

The $\overline{\sigma\phi}_0$ and μ occur to covary while performing the calibration using CoRSEER-I as discussed in Section 4.1 of the main manuscript. Here, we investigate the relationship which causes this covariance.

$$E(x) = \overline{\sigma\phi}_0 e^{-\mu x} \quad (S10)$$

$$\bar{n}(x, t) = \exp(-E(x) t) \quad (S11)$$

$$\bar{n}(x, t) = \exp(-\overline{\sigma\phi}_0 t e^{-\mu x}) \quad (S12)$$

$$\ln(-\ln(\bar{n}(x, t))) = \frac{\ln(\overline{\sigma\phi}_0 t)}{2.3} - \mu x \quad (S13)$$

This explains the same double logarithmic values of $\bar{n}$ there might be an infinite combination of $\ln(\overline{\sigma\phi}_0)$ and μ can be possible for the same luminescence depth profile and constant exposure time value; and also, there is an inverse correlation between $\ln(\overline{\sigma\phi}_0)$ and μ .

S6 References:

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