



# 1 **Simulating sedimentary burial cycles: Investigating the role of** 2 **apatite fission track annealing kinetics using synthetic data**

3  
4 Kalin T. McDannell<sup>1</sup> and Dale R. Issler<sup>2</sup>

5 <sup>1</sup>Department of Earth Sciences, Dartmouth College, Hanover NH, 03755, United States

6 <sup>2</sup>Geological Survey of Canada, Natural Resources Canada, Calgary AB, T2L 2A7, Canada

7 *Correspondence to:* Kalin T. McDannell (kalin.t.mcdannell@dartmouth.edu)

## 8 **Abstract**

9 Age dispersion is a common feature of apatite fission track (AFT) and apatite (U–Th)/He (AHe)  
10 thermochronological data and it can be attributed to multiple factors. One underappreciated and underreported cause  
11 for dispersion is variability in apatite composition and its influence on thermal annealing of fission tracks. Here we  
12 investigate, using synthetic data, how multikinetic AFT annealing behaviour (defined using the  $r_{mr0}$  parameter) can  
13 be exploited to recover more accurate, higher resolution thermal histories than are possible using conventional  
14 interpretation and modelling approaches. Our forward model simulation spans a 2 Gyr time interval with two  
15 separate heating and cooling cycles and generates synthetic AFT and AHe data for three different apatite  
16 populations with significantly different annealing kinetics. The synthetic data are used as input for inverse modelling  
17 (Bayesian QTQt model) that attempts to recover thermal history information under various scenarios. Results show  
18 that essential features of the dual peak thermal history are captured using the multikinetic AFT data alone, with or  
19 without imposed constraints. Best results are achieved when the multikinetic AFT data are combined with the AHe  
20 data (using varying  $r_{mr0}$  values from the AFT data for the He radiation damage model) and constraints are included.  
21 In contrast, a more conventional monokinetic interpretation that ignores multikinetic AFT behaviour yields incorrect  
22 thermal solutions that fail to adequately reproduce all the data. The AFT data are reproduced well but the AHe data  
23 are not. Under these conditions, incorporation of constraints can be very misleading and fail to improve model  
24 results. In general, a close fit between observed and modelled parameters is no guarantee of a robust thermal-history  
25 solution if data are incorrectly interpreted. For the case of overdispersed AFT data, it is strongly recommended that  
26 elemental data be acquired to investigate if multikinetic annealing is the cause of the age scatter. A future  
27 companion paper will explore multikinetic AFT methodology and application to detrital apatite samples from  
28 Yukon, Canada.

## 29 **1. Introduction**

30 Studies focusing on upper crustal tectonics, landscape evolution, and sedimentary basin analysis often rely on apatite  
31 fission track (AFT) and apatite (U–Th)/He (AHe) low-temperature thermochronology to decipher spatial patterns of  
32 exhumation and burial through time (e.g., Ehlers and Farley, 2003; House et al., 1998; Naeser et al., 1989; van der  
33 Beek et al., 1995; Zeitler et al., 1982). These low-temperature techniques typically produce internally consistent  
34 results in rapidly cooled, actively eroding mountain belts (e.g., Glotzbach et al., 2011), however,



35 thermochronometric harmony commonly breaks down in slowly cooled settings. There are gaps in our knowledge of  
36 how fission tracks anneal in apatite (e.g., Ketcham, 2019), how  $^4\text{He}$  diffusion occurs over geologic time (e.g.,  
37 McDannell et al., 2018), and if the mechanisms controlling these processes are fundamentally different, linked, or  
38 interact in complex and unforeseen ways. Poorly understood compound variables, both geological and analytical,  
39 sometimes yield apatite thermochronology data that are not straightforward to interpret. For example, AFT < AHe  
40 “age inversion” (e.g., Farley et al., 1996; Fitzgerald et al., 2006; Flowers and Kelley, 2011) is often encountered in  
41 continental interiors and has been attributed to the effects of slow cooling and accumulated radiation damage on He  
42 diffusion (e.g., Green et al., 2006). High age dispersion in AFT data is also seen in slowly cooled, ancient terranes  
43 (McDannell et al., 2019a), suggesting there are unexplained complexities present in both systems.

44

45 The canonical temperature sensitivity for AFT dating is ~60–125 °C (Gleadow and Duddy, 1981) and ~45–75 °C for  
46 AHe dating (Wolf et al., 1998). However, temperature sensitivity varies as a function of multiple factors such as  
47 apatite chemistry (Barbarand et al., 2003; Carlson, 1990; Crowley et al., 1990; Green et al., 1985, 1986; Ravenhurst  
48 et al., 1993) and cooling rate for AFT, and radiation damage accumulation, grain size, parent nuclide zoning, and  
49 chemistry for AHe (e.g., Djimbi et al., 2015; Farley, 2000; Gautheron et al., 2013; Gautheron et al., 2009; Recanati  
50 et al., 2017; Shuster et al., 2006). Radiation damage may also play a role in modifying apatite fission track annealing  
51 kinetics from old rocks (e.g., Carpéna et al., 1988; Hendriks and Redfield, 2005), or at least cause reduced thermal  
52 annealing resistance (McDannell et al., 2019a). This is a debated issue (Kohn et al., 2009) requiring further scrutiny  
53 and experimental work to verify empirical relationships (e.g., Carpéna and Lacout, 2010). However, observations of  
54 AHe date–U and date–elemental trends by Recanati et al. (2017) and joint AFT–AHe date–U trends by McDannell et  
55 al. (2019a) imply a complex relationship between  $\alpha$ -radiation damage and apatite chemistry, where dates increase  
56 and then decrease as a function of the estimated damage accumulated, similar to observations with zircon  
57 (Guenther et al., 2013). This suggests a change in both helium and fission-track retention at high radiation damage  
58 levels and warrants a closer inspection of apatite chemistry, radiation damage, and track annealing for applications  
59 in thermal history analysis.

60

61 We recognize chemical composition has an effect on both AFT and AHe dates, but careful investigation of this  
62 property for both chronometers remains problematic. The main factors preventing this are practical in nature, in that  
63 most AFT studies utilize compositional proxies due to ease of measurement (i.e.,  $D_{\text{par}}$  = mean etch figure width  
64 parallel to  $c$ -axis; Burtner et al., 1994; Donelick, 1993) and neglect elemental data to fully characterize samples.  
65 Likewise, the bulk AHe method is a destructive technique that precludes single-grain elemental characterization.  
66 The overwhelming majority of published studies featuring age inversion present AFT and AHe data from different  
67 grains, making direct comparisons between individual apatites challenging (Danišik, 2019). There is also the  
68 impractical comparison or statistical problem of likening AFT central ages to mean or single-grain AHe dates. The  
69 central age for AFT data is utilized to provide an approximate geometric mean age for a population of grain ages in  
70 the case of excess age dispersion (Galbraith and Laslett, 1993). Therefore, if an AFT sample fails the  $\chi^2$  test and  
71 contains discrete age components or a continuous mixture of ages (Galbraith and Green, 1990; Galbraith and Laslett,



72 1993), then the meaning of the central age is somewhat misleading for comparative purposes. The same applies to  
73 averaged AHe dates if accumulated radiation damage varies between grains.

74

75 In the overall context of age scatter, another equally viable possibility is that overdispersed or inverted dates for the  
76 AFT and AHe thermochronometers occur as a result of variable intrasample retentivity (i.e., resistance to track  
77 annealing and diffusive He loss) for both systems due to the effects of apatite chemical composition. In the simple  
78 case one can imagine two apatite grains, a fluorapatite and the other a more retentive chlorapatite, where the former  
79 is dated by FT and the latter is dated by (U–Th)/He. Assuming a slow-cooling history, the grain dated by AFT may  
80 yield a date that is younger than the grain dated by the (U–Th)/He method solely due to compositional differences.  
81 Here, we present simple examples demonstrating these effects using synthetic AFT and AHe data derived from  
82 forward models utilizing the  $r_{mr0}$  kinetic parameter based on apatite composition (Carlson et al., 1999; Ketcham et  
83 al., 1999). The synthetic data are exaggerated, implementing extreme endmember kinetics that are rare, but not  
84 unheard of, in natural crystalline basement samples and more commonly encountered in detrital samples. This was  
85 done to illustrate that multikinetic AFT samples provide an expanded range in thermal sensitivity and that AFT data  
86 may be misrepresented (under the assumption that the central age is wholly descriptive of a sample) if potential  
87 kinetic sub-populations governed by composition are not accounted for during data interpretation or kinetic proxy  
88 data are imprecise (i.e.,  $D_{par}$ ; Issler et al., 2018; Schneider and Issler, 2019). Analogously, in the absence of  
89 retentivity information for the AHe system, using a default “fluorapatite” value may completely misrepresent a  
90 sample by introducing modelling artifacts that distort time-temperature ( $t$ – $T$ ) solutions, or even prevent viable  $t$ – $T$   
91 paths from being found during thermal history analysis. These exercises were performed assuming that we knew the  
92 true thermal history, which is almost always not the case, and they are meant to encourage users of  
93 thermochronology data to more thoroughly interpret data and explore kinetic models before undertaking thermal  
94 history simulations. The results in this paper give us confidence in our treatment of real data and support the idea  
95 that the multikinetic AFT method yields higher resolution thermal histories than the conventional method. In a  
96 future companion paper, we will specifically discuss elemental data collection, multikinetic workflow and  
97 interpretation schemes, and thermal history analysis of natural detrital samples from Yukon, Canada.

## 98 **2. Apatite chemistry, track annealing, and the experimentally derived $r_{mr0}$ parameter**

99 The empirical  $r_{mr0}$  kinetic parameter was derived by characterizing track annealing with respect to chemical  
100 composition (Carlson et al., 1999) to produce a multikinetic annealing model (Ketcham et al., 1999). Later work  
101 updated the equation and annealing data fits (Ketcham et al., 2007) by combining the dataset of Barbarand et al.  
102 (2003) with the 1999 dataset. However, the later reformulation of  $r_{mr0}$  is different due to the dominant influence of  
103 Cl and OH (and generally lower cation concentrations) in the 2003 dataset, which considerably changes the fitting  
104 parameterization. Although the  $r_{mr0}$  kinetic model shows you can reconcile the experimental annealing data with  
105 apatite composition, this does not necessarily mean that more data equates to a better calibration. More data changes  
106 the calibration, but “improvement” depends on whether the calibration data are representative of the natural range of  
107 apatite compositions or are skewed to a particular composition. The Ketcham et al. (2007) model still suffers from



108 an uneven distribution of data and includes a subset of possible compositional ranges that cause the revised equation  
109 to narrow the range of  $r_{mr0}$  slightly from the original model. In our view, the 2007 multikinetic model is no better or  
110 worse than the original model, however the 1999 model is less dominated by chlorapatite compositions, which aids  
111 in clearer multikinetic interpretation (i.e., less kinetic population overlap) for natural AFT samples. It is a reasonable  
112 assumption that the same annealing mechanism and therefore the same kinetic formulation applies to all apatite  
113 varieties. It is primarily our lack of knowledge regarding composition and the relation with annealing, not  
114 necessarily erroneous models that are the main issue for kinetic model calibration. Nevertheless, the utility of the  
115  $r_{mr0}$  function remains for explaining overall apatite annealing–compositional trends. Here we review the  $r_{mr0}$   
116 parameter in the context of the original Carlson et al. (1999) expression. The reader is referred to the original papers  
117 or Ketcham (2019) for a comprehensive discussion of  $r_{mr0}$ .

118  
119 The  $r_{mr0}$  value comes from a simple normalization function that relates one apatite to another for the purpose of  
120 comparing annealing behaviour at laboratory timescales, using the equation:

$$122 \quad r_{lr} = \left( \frac{r_{mr} - r_{mr0}}{1 - r_{mr0}} \right)^k \quad (1)$$

123  
124 Where  $r_{lr}$  and  $r_{mr}$  are the reduced lengths of the apatite that are less resistant and more resistant to annealing,  
125 respectively and  $r_{mr0}$  and  $k$  are fitted parameters. Specifically,  $r_{mr0}$  is the reduced fission-track length of the more  
126 resistant apatite at the point in time and temperature where the less resistant apatite is totally annealed, allowing a  
127 direct comparison between any two apatites (Ketcham et al., 1999). Ketcham et al. (1999) used B2 apatite from  
128 Bamble, Norway (highly enriched in Cl and OH) as the reference datum for  $r_{mr0}$ , since this was the apatite most  
129 resistant to annealing in the Carlson et al. (1999) experiments. Therefore,  $r_{mr0}$  values approaching one, signify lower  
130 retentivity, whereas those approaching zero are more retentive, with common fluorapatite defined by a  $r_{mr0}$  value of  
131 0.84. Individual  $r_{mr0}$  fits of apatite pairs revealed overall good agreement between measured and predicted mean  
132 (and c-axis projected) lengths, however the simultaneous fits to the entire apatite dataset were lower quality. The  
133 poorer fit was perhaps due to subtle differences in etching/annealing conditions (i.e., temperature control), the  
134 simplification that  $r_{mr0} + k \approx 1$ , or insufficient compositional diversity and/or elemental data. For example, Si was  
135 not accounted for in the Carlson et al. (1999) and Ketcham et al. (1999) studies but a subsequent study by Tello et al.  
136 (2006) found that Itambé apatite was more resistant to annealing than the Durango apatite laboratory age standard  
137 and is nearly 13x richer in Si (4.15 wt.% Si; simultaneous 1999 fit  $r_{mr0} = 0.819$  excluding Si). Comparing Itambé to  
138 Durango implies higher retentivity for the former, yet the difference in  $r_{mr0}$  between Itambé and Durango is very  
139 small using the 1999  $r_{mr0}$  equation (0.31 wt. % Si; simultaneous 1999 fit  $r_{mr0} = 0.827$ ). The  $r_{mr0}$  value for Itambé  
140 calculated using the Ketcham et al. (2007) equation is 0.785; suggesting track retentivity is greater than Durango,  
141 although the 2007 equation is biased towards more retentive apatite. These differences are just one example  
142 indicating further annealing studies are required to account for unusual elemental substitutions that nonlinearly  
143 influence annealing behaviour at the cation sites in apatite (Barbarand et al., 2003; Carlson et al., 1999; Ketcham et  
144 al., 2007).



145

146 We utilize the relationship established between  $r_{mr0}$  and measured Cl to calculate an “effective Cl” (eCl) value in  
147 atom per formula unit (apfu) from collected electron microprobe data (see McDannell et al., 2019b for further  
148 explanation). Effective Cl is the Cl concentration required to yield an equivalent  $r_{mr0}$  value for the Ketcham et al.  
149 (1999) annealing model based on the published correlation between Cl and  $r_{mr0}$  in Carlson et al. (1999). The eCl  
150 value (e.g., Issler et al., 2018; McDannell et al., 2019b) is used to transform the nonlinear  $r_{mr0}$  parameter to a linear  
151 form for data interpretation using the equation (given in figure 7 of Ketcham et al., 1999):

152

$$153 \quad r_{mr0} = 1 - \exp [2.107(1 - Cl) - 1.834] \quad (2)$$

154

155 In addition, the Ketcham et al. (1999) expression relating  $r_{mr0}$  to  $D_{par}$  is:

156

$$157 \quad r_{mr0} = 1 - \exp [0.647(D_{par} - 1.75) - 1.834] \quad (3)$$

158

159 The constants in these equations changed slightly in the Ketcham et al. (2007) multikinetic model revision but  
160 remain similar to the original calculations. Equations (2) and (3) allow the transformation between measured kinetic  
161 parameters (i.e.,  $D_{par}$  and Cl) to  $r_{mr0}$  and vice versa. For example, the  $r_{mr0}$  value from the Ketcham et al. (2007) model  
162 is 0.83 for fluorapatite, which translates to an eCl value of ~0.03 apfu and an eDpar of ~1.85  $\mu\text{m}$ .

163

164 Fission-track kinetics have also been used to describe changes in  $^4\text{He}$  diffusivity in the apatite (U–Th)/He system  
165 (e.g. Flowers et al., 2009). The development of a model to explain radiation damage effects on He diffusivity  
166 (Shuster and Farley, 2009; Shuster et al., 2006) resulted in the radiation damage accumulation and annealing model  
167 (RDAAM) by using fission-track annealing kinetics of Ketcham et al. (2007) as a proxy for  $\alpha$ -damage or bulk  
168 radiation damage annealing (Flowers et al., 2009). The fundamental assumption being that  $\alpha$ -damage and fission-  
169 track damage anneal at the same rate, enabling the use of the  $r_{mr0}$  parameter in the RDAAM, set to typical  
170 fluorapatite kinetics ( $r_{mr0} = 0.83$ ). This allows a comparison between fission track and He data within the same  
171 kinetic framework. However, there is an apparent divergence in damage kinetics, and we now understand that  
172 assuming similar annealing kinetics is an oversimplification and probably incorrect, especially when rocks reside for  
173 long intervals at low temperatures  $<100^\circ\text{C}$  and at high levels of accumulated radiation damage (Fox and Shuster,  
174 2014; Gautheron et al., 2013; Ketcham, 2019; Ketcham et al., 2017; McDannell et al., 2019a; Recanati et al., 2017;  
175 Willett et al., 2017). However, for common low-damage (i.e., low U) apatite and certain thermal histories, the  
176 kinetics remain valid to first order. We use the  $r_{mr0}$  parameter to examine the relationship between apatite  
177 composition and track retentivity (and He diffusivity) and how accounting for or overlooking these associations  
178 influence data interpretation, and ultimately, thermal history modelling results.

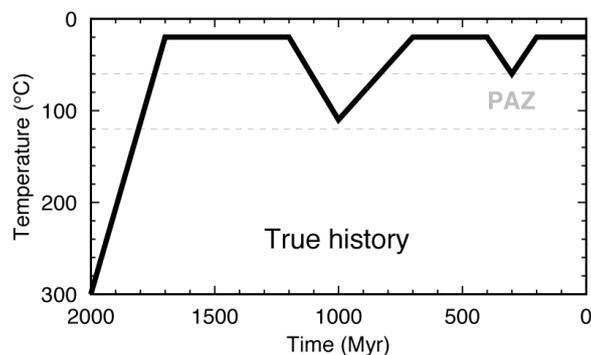


179 **3. Forward and inverse modelling of multikinetic synthetic data**

180 **3.1 Forward modelled synthetic AFT and AHe data from a predetermined thermal history**

181 Synthetic AFT data were generated from forward modelling a two-pulse heating history over 2000 Myr using the  
182 QTQt software v. 5.7.3 (Gallagher, 2012) implementing Ketcham et al. (1999) annealing kinetics (fig. 1), with one  
183 maximum heating event occurring at 1000 Ma (110°C) and the other at 300 Ma (60°C). AFT ages and track length  
184 data (fig. 2) were randomly predicted for three kinetic populations as external detector method (EDM) data in QTQt.  
185 We specified three AFT kinetic populations of 10 age grains each, increasing in retentivity with  $r_{mr0}$  values of 0.882  
186 ( $eCl = -0.144$  apfu), 0.820 ( $eCl = 0.057$  apfu), and 0.263 ( $eCl = 0.726$  apfu) using individual-fit  $c$ -axis projected  
187 length kinetic data for distinct apatites from Ketcham et al. (1999). Population one is set to the Holly Springs  
188 (Georgia, USA) hydroxyapatite  $r_{mr0}$  that typifies the lowest calculated retentivity in the Carlson et al. (1999) dataset,  
189 population two uses Durango apatite kinetics (laboratory age standard), whereas population three is set to Tioga  
190 (Pennsylvania, USA) Fe-Cl apatite, which is characterized by high retentivity and is an outlier of the Carlson et al.  
191  $r_{mr0}$ -fitting dataset. The specified thermal history produced three AFT model ages of 670 Ma, 843 Ma, and 1602 Ma  
192 (fig. 2). Seventy-five tracks were generated for each kinetic population with mean  $c$ -axis projected track lengths  
193 (MTL) of  $13.32 \pm 1.33$   $\mu\text{m}$  ( $1\sigma$ ),  $14.24 \pm 1.42$   $\mu\text{m}$ , and  $14.65 \pm 1.47$   $\mu\text{m}$ , respectively. The initial (pre-annealed)  
194 track lengths ( $l_{oc}$ ) for each kinetic population were calculated as 16.17  $\mu\text{m}$ , 16.40  $\mu\text{m}$ , and 17.16  $\mu\text{m}$  with increasing  
195 retentivity and were estimated from the equivalent  $D_{par}$  calculated from the indicated  $r_{mr0}$  value for each kinetic  
196 population (equation 3 above) using the  $l_{oc}$ - $D_{par}$  relation from Carlson et al. (1999). Three AHe dates were also  
197 forward modelled using the radiation damage accumulation and annealing model (RDAAM) of Flowers et al.  
198 (2009), which implements the Ketcham et al. (2007) kinetics for radiation damage annealing. We applied Holly  
199 Springs, typical endmember fluorapatite ( $r_{mr0} = 0.83$  and the RDAAM default), and Tioga apatite  $r_{mr0}$  values to AHe  
200 grains, all with spherical grain radii of 50  $\mu\text{m}$  and 25 ppm U (Th and Sm discounted for simplicity). The uncorrected  
201 AHe dates ( $\alpha$  ejection-corrected date in brackets) were 585 Ma [813 Ma], 610 Ma [848 Ma], and 819 Ma [1139 Ma]  
202 predicted using the same  $t$ - $T$  history (fig. 1) as the AFT data.

203



204

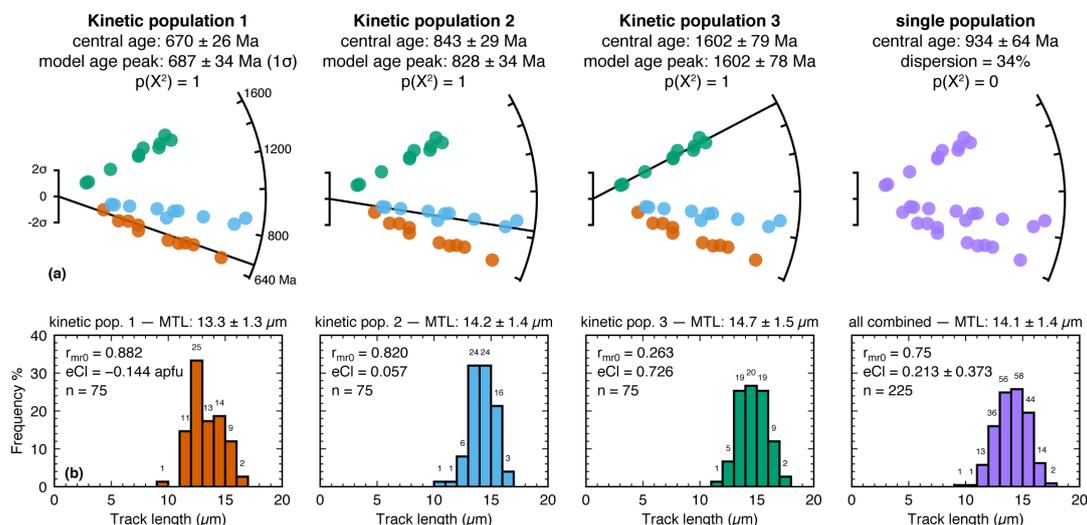
205 **Figure 1:** Thermal history used to predict synthetic AFT and AHe data. This  $t$ - $T$  path is referred to as the “true” thermal history  
206 throughout this paper. The predicted synthetic data were then used as input for QTQt to recover the thermal history through  
207 inverse modelling. PAZ = partial annealing zone for fission tracks.



### 208 3.2 Methods for inverting AFT and AHe synthetic data for thermal history

209 We attempted to recover the true thermal history used to predict the synthetic data from Sect. 3.1 using the QTQt  
 210 software. These exercises imitate real thermal history investigation in the context of incomplete geologic  
 211 knowledge, complex or imperfect datasets, and judgement calls that are typically made by researchers implementing  
 212 thermochronology data and performing modelling to infer quantitative information about geologic processes. We  
 213 also explore the effects of kinetic assumptions for AHe dates or the consequences of neglecting the identification of  
 214 multikinetetic populations during AFT modelling. An important point is that QTQt will generate thermal histories  
 215 regardless of feasibility, and it is up to the user to understand the ramifications of this and make sensible decisions  
 216 about modelling input and output (Gallagher and Ketcham, 2018; Vermeesch and Tian, 2014). We used QTQt  
 217 because it is sensitive to the number and quality of data during history inference (i.e., notionally improving model  
 218 results with additional, high quality data) and specifically because it *will* generate model histories regardless of the  
 219 physical or geologic plausibility for a history simulation — this was done to explore the possible effects of improper  
 220 data treatment or misinterpretation.

221



222

223 **Figure 2:** Predicted synthetic AFT data from the thermal history in figure 1. Multikinetetic age populations were individually  
 224 predicted using distinct  $r_{\text{mr}0}$  kinetics shown in (B) panels (discussed in the text). These data were then input in QTQt and inverted  
 225 in an attempt to recover the true thermal history in figure 1 (see fig. 3). (A) Central age and  $1\sigma$  errors are indicated for each  
 226 kinetic population. Kinetic populations one, two, and three are displayed as arms on their respective radial plots, with individual  
 227 AFT ages closer to the origin being less precise. The last radial plot shows all thirty individual grains and demonstrates that when  
 228 taken together, the combined sample fails the  $\chi^2$  test ( $p < 0.05$ ) for homogeneity (i.e., that all grains belong to a single underlying  
 229 age population) suggesting multiple age populations. This is the scenario most researchers would start with before evaluating the  
 230 sample for potential multikinetetic behaviour. Mixture modelling was subsequently performed on the combined sample and the  
 231 model age peaks that were picked seamlessly align with the individual kinetic population central ages. This aligns with how  
 232 populations would be defined and compared with the elemental chemistry for individual age grains during multikinetetic  
 233 interpretation. (B) The predicted track length distributions for each kinetic population from the thermal history in Figure 1 using  
 234 the specified kinetic parameter value. The last panel on the right combines all tracks from each kinetic population. Numbers on  
 235 the histogram are the number of tracks in each  $\mu\text{m}$  bin. Abbreviations: eCl = effective Cl; MTL = mean track length.



236 The  $r_{m0}$  values for AFT and AHe data were held fixed for simulations and an appropriate level of noise was added  
237 to the synthetic dataset by adding age scatter to AFT dates and setting typical uncertainties for predicted AHe dates  
238 (all information given in ascending retentivity/kinetic population order). The AFT data were recast from QTQt  
239 individual synthetic output files using random spontaneous/induced track ( $N_s/N_i$ ) ratios that produced central ages  
240 for each kinetic group that were in agreement with forward model predictions using identical EDM parameters with  
241 a  $\zeta$ -calibration value =  $350 \text{ yr cm}^{-2}$ , induced track density ( $\rho_{Di}$ ) =  $2.5 \times 10^6 \text{ cm}^{-2}$ , and dosimeter tracks ( $N_d$ ) = 10000.  
242 These common values made it so each population was simulated as being from the same grain mount for the  
243 purposes of easy comparison and  $t$ - $T$  inversion. Population one central age was calculated as:  $670 \pm 26 \text{ Ma}$ ,  
244 population two was calculated as:  $843 \pm 29 \text{ Ma}$ , and population three was calculated as:  $1602 \pm 79 \text{ Ma}$ . The  
245 synthetic AFT sample has an overall central age of  $934 \pm 64 \text{ Ma}$  ( $1\sigma$ ,  $X^2 = 0.0$ , MSWD = 9, 34% dispersion,  $n = 30$ )  
246 when all age grains are combined. Three mixture model age peaks of  $687 \pm 34 \text{ Ma}$ ,  $828 \pm 34 \text{ Ma}$ , and  $1602 \pm 78 \text{ Ma}$   
247 ( $1\sigma$ ) were selected in IsoplotR (Vermeesch, 2018) for the combined AFT data, which are in agreement with the  
248 individual kinetic population central ages. The uncorrected AHe dates used all default RDAAM settings with the  
249 exception of  $r_{m0}$  and the dates were input as:  $585 \pm 17 \text{ Ma}$ ,  $610 \pm 18 \text{ Ma}$ , and  $819 \pm 25 \text{ Ma}$  (all 3% errors,  $1\sigma$ ).

250

251 We ran QTQt in multiple stages to tune Bayesian sampling and to ensure the acceptance rates for time and  
252 temperature were between  $\sim 0.1$ – $0.7$ , within the acceptable limits discussed in Gallagher (2012). Inversions were run  
253 for  $>500,000$  to  $>1,000,000$  total iterations (burn-in and post-burn-in) and were considered complete when the  
254 likelihood distribution was stationary (i.e., there was no trend in the likelihood values with a stable or “flat” mean;  
255 Gallagher, 2012). The modelling  $t$ - $T$  space (prior) was designated as  $1000 \pm 1000 \text{ Ma}$  and  $150 \pm 150^\circ\text{C}$  with a  
256 maximum allowed heating/cooling rate of  $5^\circ\text{C}/\text{Myr}$ . Sampling proposed outside of the prior was prevented and more  
257 complex models were rejected. Therefore,  $t$ - $T$  points were only added if they provided a better fit to the input data.  
258 The long time interval for these model inversions are styled after a typical cratonic history and the only constraint  
259 that was consistently enforced was starting the model at  $300 \pm 1^\circ\text{C}$  at  $2000 \pm 1 \text{ Ma}$ . For our purposes, this scenario is  
260 considered a “no constraint” model, since we apply this as a starting condition for all inverse models well above the  
261 sensitivity of our thermochronology data. We also ran models that enforced constraint boxes (i.e., with either one or  
262 two boxes) at  $20 \pm 10^\circ\text{C}$  at  $1650 \pm 100 \text{ Ma}$  and  $20 \pm 10^\circ\text{C}$  at  $500 \pm 50 \text{ Ma}$ , requiring  $t$ - $T$  paths to pass through them.  
263 These  $t$ - $T$  boxes were treated as “known” geologic information for the inversions. For all models presented  
264 hereafter, we show the QTQt Maximum Likelihood (ML; i.e., more complex, best fit  $t$ - $T$  path to the observed data,  
265 coloured line) and Expected models (EX; i.e.,  $\sim$ weighted mean  $\pm$  95% credible interval; long dashed line and gray  
266 envelope) with respect to the true thermal history used to predict the synthetic data (fig. 1). In Bayesian inference,  
267 the posterior probability is proportional to the likelihood multiplied by the prior, and in QTQt the prior acts as a  
268 penalty against making the model too complex and thus the Maximum Posterior (MP) model will be the simpler  $t$ - $T$   
269 path when compared to the ML path (i.e., typically fewer  $t$ - $T$  points; Gallagher, 2012). We have excluded the MP  
270 model for plot clarity for most output because the ML and MP paths are identical or nearly so for most scenarios,  
271 which implies a well sampled and constrained ensemble of solutions (Gallagher and Ketcham, 2020).



#### 272 4. Model inversion results

273 QTQt inversion results are shown in figure 3 and examine the implications of multikinetic AFT, joint models with  
274 multikinetic AFT and AHe grains using the correct kinetics (i.e., the kinetics implemented during forward modelling  
275 to predict AHe dates), and different combinations of incorrect monokinetic AFT models where the three multikinetic  
276 populations were combined and treated as a single AFT sample and/or AHe dates were assumed to be the  
277 endmember fluorapatite  $r_{mr0}$  value. Figure 4 depicts the results comparing observed synthetic data and model  
278 predictions for the inversions in figure 3. The first three models are “multikinetic AFT only” models (fig. 3A–C),  
279 whereas the second row of models depicts results for three multikinetic AFT populations and three AHe grains (Fig.  
280 3D–F). The last three panels are the single population AFT models (fig. 3G–I). We prevented t–T points from being  
281 added during QTQt inversions unless the addition of points provided better agreement between observed and  
282 predicted data. Therefore, all of our preferred results and discussion focus on the Maximum Likelihood model t–T  
283 path, yet we show the Expected model and 95% credible interval for comparison and to provide a general picture of  
284 the overall model ensemble. It should be noted that because the EX model undergoes a simple temperature  
285 weighting in QTQt, the upper 95% credible interval will almost always be biased to slightly cooler temperatures  
286 than if an exponential temperature weighting were to be applied that preferentially weights higher temperatures.

#### 287 4.1 AFT-only models – identified multikinetic age populations and correct kinetics

288 The first model was setup to simultaneously invert each AFT kinetic population without AHe data for scenarios with  
289 a “no constraint” model, a “single t–T constraint” model, and “two t–T constraints” model (fig. 3A–C). These  
290 simulations were meant to be the ideal case using a lone AFT chronometer with extended thermal sensitivity due to  
291 the presence of multikinetic apatite populations. We investigated the ability of QTQt to recover the true thermal  
292 history using properly identified kinetic age populations while utilizing the true  $r_{mr0}$  value from forward modelling  
293 for each population under varying degrees of geologic assumptions or constraints. The general shape, timing, and  
294 magnitude of the true history form and peak temperatures are recovered for the multikinetic AFT models regardless  
295 of whether or not constraint boxes were used. This suggests to us that the combination of high-quality, distinct age  
296 and length populations enhance t–T history resolving power, which becomes progressively improved if kinetic  
297 populations sample a broad range of kinetic space (predicted AFT parameters closely agree with the synthetic data;  
298 fig. 4A–C).

#### 299 4.2 AFT + AHe models – consequences of the $r_{mr0}$ parameter

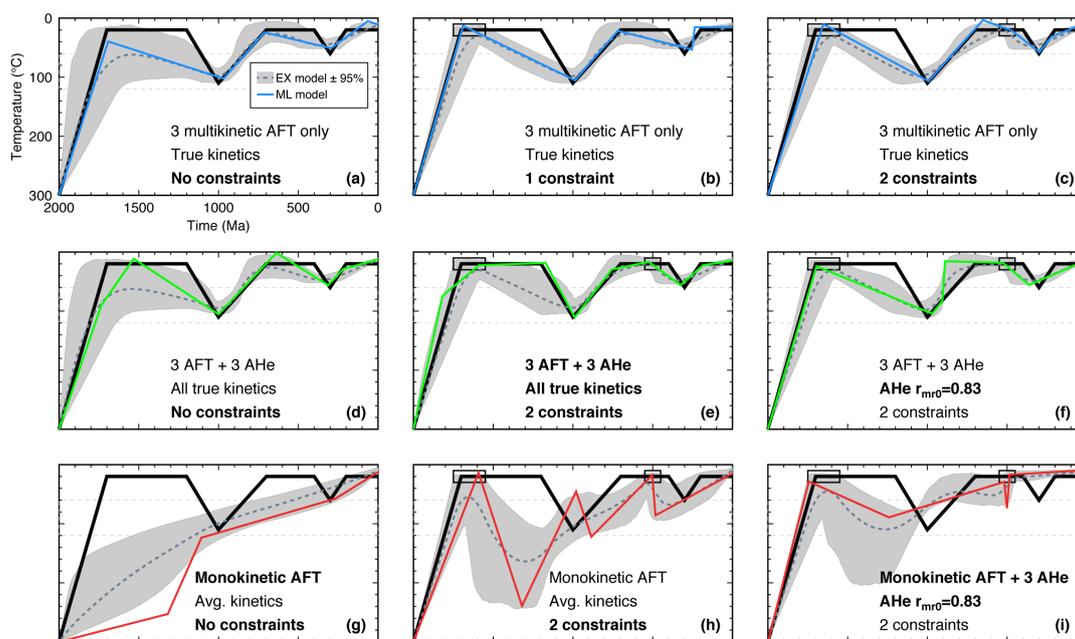
300 The addition of the three AHe dates using their *correct* kinetics (i.e.,  $r_{mr0}$  values) along with the three multikinetic  
301 AFT populations (fig. 3D) improved thermal history recovery with respect to the AFT-only models (fig. 3A–C),  
302 while the addition of two constraint boxes produced a ML model t–T path that reproduced nearly all features of the  
303 true thermal history (fig. 3E). Figure 3E is the best thermal history model that utilized all assumptions and  
304 information used during forward model generation of the synthetic dataset and provides the closest fit to the  
305 synthetic data (fig. 4E). Setting all three AHe grains to  $0.83 r_{mr0}$  produces distortion of the model history with respect  
306 to the true history (fig. 3F). The model predicts three AHe dates that are virtually identical but provide a poor fit to



307 the input synthetic AHe ages (fig. 4F). The 610 Ma AHe grain (true kinetic  $r_{mr0}$  value = 0.83) was on the margin of  
308 acceptability.

#### 309 **4.3 Monokinetic AFT models – incorrectly combined kinetic populations**

310 In our experience, multikinetic behaviour is not uncommon for basement samples characterized by complicated  
311 burial histories and nearly always present for detrital apatite samples derived from complex source areas that  
312 experience multiple heating events. In our “monokinetic” scenario, the multikinetic AFT data were incorrectly  
313 treated as a single population and modelled using the central age, MTL, and average eCl or  $r_{mr0} \pm 1\sigma$  of the *entire*  
314 *pool* of synthetic single-grain ages. As previously mentioned, combining the three populations caused the sample to  
315 fail the chi-square test ( $X^2 = 0.0$ ) and the calculated AFT central age was  $934 \pm 64$  Ma, the overall MTL was  $14.07 \pm$   
316  $1.40 \mu\text{m}$  ( $n = 225$ ), and the average eCl is  $0.213 \pm 0.373$  apfu (equivalent  $r_{mr0} \approx 0.75$ ) for all grains. AFT data are  
317 usually treated as such in the published literature and overdispersed data are often modelled regardless of  $\chi^2$   
318 statistics. This situation could conceivably occur when the three kinetic populations were either ignored or there was  
319 insufficient kinetic parameter resolution to identify discrete kinetic groups. A sample could also simply not be  
320 multikinetic — but the models here are meant to illustrate the hazards of monokinetic misinterpretation for thermal  
321 history analysis. In the monokinetic simulation without constraints, both the ML and EX t–T paths do not accurately  
322 reproduce the true thermal history (fig. 3G). In this instance the ML path passes directly through both true  
323 Phanerozoic thermal maxima and yields excellent fits to the observed synthetic data (fig. 4G). The addition of two  
324 constraint boxes produced even more complex and highly inaccurate t–T solutions (fig. 3H), yet well reproduce the  
325 observed AFT data (fig. 4H). The AFT sample was modelled as monokinetic again (fig. 3I), but also included the  
326 three AHe dates using uniformly applied default RDAAM  $r_{mr0}$  value of 0.83 for each apatite grain to provide further  
327 insight into whether this combination could yield a better outcome just from the addition of more data for the  
328 inversion. The EX model is still inaccurate but the addition of AHe grains made the ML path simpler, nevertheless it  
329 is still distorted and poorly reproduces the true thermal history. QTQt also failed to accurately reproduce the true  
330 AHe dates and predicted the same date for all three grains (fig. 4I). This may be because the second 610 Ma AHe  
331 grain utilized the true  $r_{mr0}$  value of 0.83 from the forward modelling and was the best-predicted date of the three  
332 (close to the observed date upper uncertainty limit) and dominated the iterative sampling during the inversion. The  
333 AHe kinetics produced forward model dates that were distinctly older (819 Ma) and younger (585 Ma) than the  
334 (middle) 610 Ma grain but these were unable to be reproduced by the inverse model assuming incorrect  $r_{mr0}$  kinetics.  
335



336

337 **Figure 3:** Thermal history inversion results from QTQt under different imposed kinetic and t-T assumptions. (A–C) show the  
338 “AFT only” models that utilized three multikinetik AFT populations (discussed in the text) as the only input data. The true  $r_{m0}$   
339 kinetics applied during forward modelling were entered in the input files and held fixed for each kinetic population during the  
340 inversion. (D–E) show the results of models that correctly utilized three multikinetik AFT kinetic populations and three AHe  
341 dates all with the true kinetics held fixed. Panel E is the best model inversion incorporating all correct thermochronometer  
342 information used during forward modelling of the synthetic data set. The panel (F) model was completed under the same  
343 conditions as panels (D–E) except that the three AHe grains all employ the incorrect (in the oldest and youngest cases) RDAAM  
344 default fluorapatite  $r_{m0}$  value of 0.83 as the kinetic parameter. Panels (G–I) were modelled assuming a “monokinetic” or  
345 traditional single population AFT sample that combines all three multikinetik populations into one. For all panels: Thick black  
346 line is the “true” thermal history from figure 1; coloured, solid lines are the Maximum Likelihood model (best fit) t–T path from  
347 QTQt; dashed gray lines are the Expected model t–T path with light gray 95% credible interval envelope. Assumed t–T  
348 constraints are black boxes that require thermal histories to pass through them during the inversion.

## 349 5. Discussion

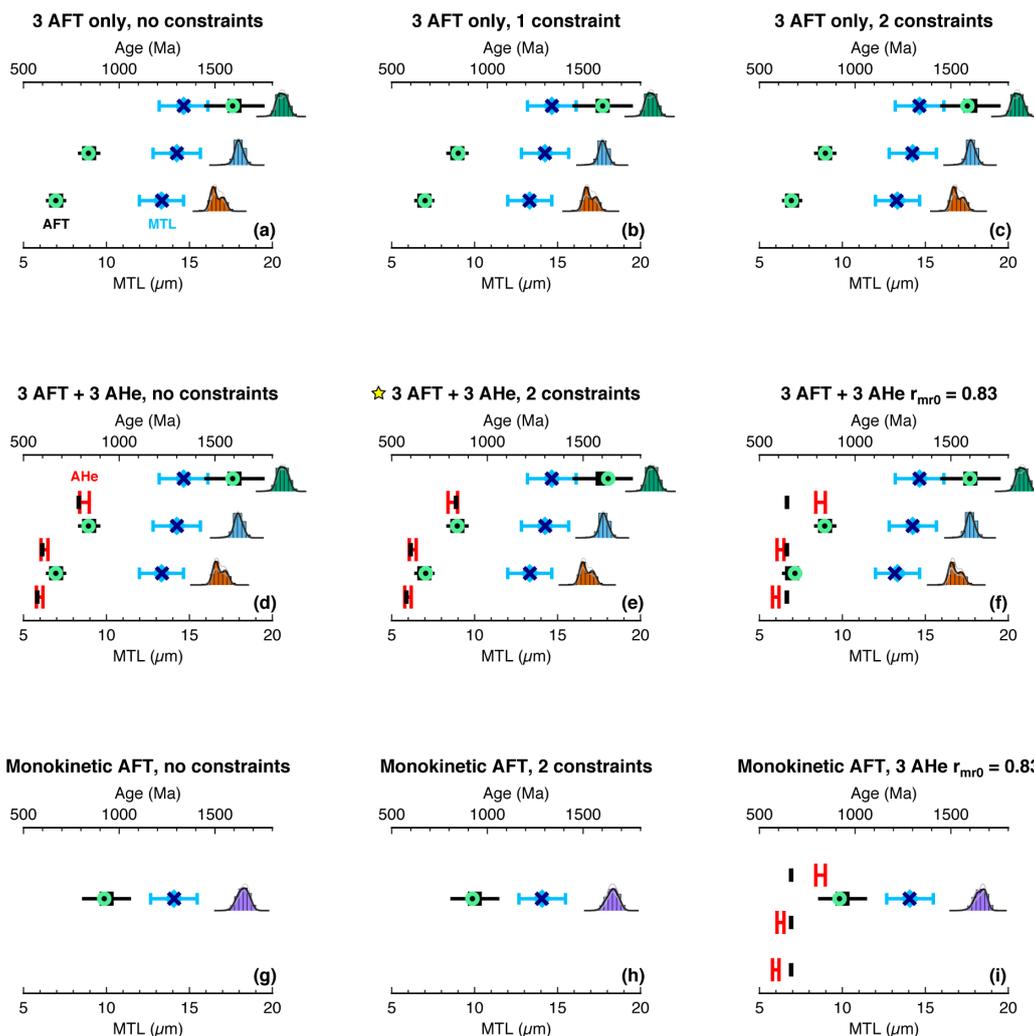
### 350 5.1 Apatite composition and multikinetik interpretation

351 The AFT and AHe modelling results presented here may seem intuitive based on the implemented kinetics and  
352 modelling exercises using synthetic data, but are worth discussing nonetheless, since situations where highly  
353 variable apatite compositions could influence thermochronometric dates are likely to be encountered in natural  
354 samples. The results shown here indicate the benefits offered by interpreting intrasample AFT kinetic populations  
355 for inverse modelling and also show how inappropriate assumptions regarding kinetic parameters can greatly  
356 influence model outcome. Our examples were determined for a single, distinct thermal history, and yet they  
357 establish that apatite composition and multikinetik interpretation (when appropriate) provide valuable information  
358 for thermal history modelling — and are mostly unexplored, or at least underutilized by routine AFT studies.  
359 Gallagher and Ketcham (2020) also touch on these points in response to the lengthy modelling discussion sparked  
360 by Vermeesch and Tian (2014) and are primary themes of this work.

361



362 Collection of elemental data and interpretation of multikinetic samples is particularly important for providing greater  
363 t–T resolution (fig. 3A–F), whereas combining or overlooking kinetic populations effectively smears the t–T signal  
364 contained in the individual kinetic groups and produces a meaningless hybrid thermal history model (fig. 3G–I). We  
365 could disregard these incorrect model simulations as self-fulfilling due to forward modelling a synthetic dataset and  
366 assuming “perfect” kinetic models, however for real scenarios we would not know the true thermal history and  
367 without other information, this class of results could be interpreted as geologically meaningful. Perhaps more  
368 important are the broader implications for thermal history modelling if there are inappropriate assumptions  
369 regarding data interpretation and certain steps are not taken to fully evaluate multikinetic AFT samples (fig. 3G–I),  
370 especially at longer timescales where there is greater uncertainty and less geologic control. An important point is  
371 that if multikinetic populations exist and are properly interpreted, they have the potential to constrain a much  
372 broader range of t–T space than an incorrect monokinetic (single population) interpretation for an overdispersed  
373 AFT sample.  
374



375

376 **Figure 4:** QTQt inversion predictions compared to “observed” synthetic thermochronology data generated during forward  
 377 modelling. Panel letters correspond to counterpart t–T model panels in figure 3. All predictions are for the Maximum Likelihood  
 378 models. Squares are observed AFT central age  $\pm 2\sigma$ , circles are predicted AFT age, diamonds are observed MTL  $\pm 1\sigma$ , and X-  
 379 symbols are the predicted MTL. Individual model fits to each track length distribution for the AFT kinetic populations are also  
 380 shown and color-coded the same as figure 2. Observed apatite He dates shown by red H-symbol (spans the  $1\sigma$  error range quoted in  
 381 the text) and predicted AHe dates are black bars. Panel E with star is our best model that accounts for all multikinetick AFT  
 382 populations and utilizes the true AHe kinetics and two geologic constraints, all combined for the highest thermal history  
 383 resolution. Note: track length distributions are arbitrarily placed next to their respective age population and were not plotted with  
 384 respect to the MTL plot axis.

### 385 5.2 Data quality and kinetic parameter influence on t–T resolution

386 The overall temporal and thermal resolution contained in multikinetick AFT data is influenced by multiple factors  
 387 such as, the amount and distribution of the data (i.e., if the majority of the data are contained in one population  
 388 versus distributed more equally), thermal history (i.e., the magnitude and sequence of heating-cooling events), and  
 389 kinetics (i.e., the range of temperature sensitivity). A greater number of different kinetic groups are sensitive to more



390 extensive parts of the thermal history than a single population. However, the ability to recover thermal history  
391 information depends on the details of the thermal history; if maximum temperatures occur late in the history then  
392 previous events are thermally overprinted and the early history is obscured or erased entirely. We intentionally use  
393 an ideal synthetic dataset with well-defined kinetic populations that have an equal distribution of data across all  
394 populations. Natural populations may have an uneven distribution of grains and therefore populations that contain  
395 the most data will best resolve distinct parts of the thermal history. Our QTQt inversions demonstrate the ability of  
396 these data to inform  $t$ - $T$  modelling in the context of variable kinetics and different modeller assumptions. The  
397 similarity between Expected models that do and do not require paths to pass through explicit  $t$ - $T$  boxes (e.g., fig.  
398 3A–C) is informative for general modelling practices using Bayesian methods. This tells us that the multikinetic data  
399 being inverted have enough sensitivity to resolve the general  $t$ - $T$  history without requiring explicit conditions  
400 imposed on the  $t$ - $T$  search. This is perhaps unexpected, as the Bayesian sampling implemented by QTQt generally  
401 favours simpler models over complex ones, which is a possible deterrent for users investigating deep-time thermal  
402 histories (McDannell and Flowers, 2020). However, this should not preclude the use of QTQt for deep-time  
403 problems, as the addition of thermochronological data augments inferences regarding thermal-history complexity in  
404 QTQt.

405

406 On the other hand, enforcing constraints while utilizing fewer chronometers and ignoring data complexity or kinetic  
407 trends works against us. The main region of  $t$ - $T$  space that proved difficult to resolve in all models was the  
408 prolonged periods at low temperature. This was anticipated since the kinetic models and chronometers themselves  
409 are rather insensitive to temperatures  $< 50^{\circ}\text{C}$ . An interesting outcome is that for the “AFT only” models (fig. 3A–C)  
410 the 95% credible interval around the EX  $t$ - $T$  path gained precision at the expense of accuracy when constraint boxes  
411 were added (fig. 3B–C), whereas the model without geologic constraints was less precise but more accurate at the  
412 95% level (fig. 3A). It should be noted that since we penalized unnecessary complexity as an explicit condition on  
413 the model prior, there were individual paths (not shown) that were more similar to the true history for these three  
414 simulations, yet QTQt considered these solutions lower relative likelihood. We may expect this compromise  
415 between accuracy (i.e., closer to the true solution) and precision (i.e., greater uncertainty) because subsequent  
416 heating event(s) erase  $t$ - $T$  information and the earlier or older, low-temperature parts of the history will be less and  
417 less resolvable with additional reheating and thus may require constraint boxes to assist in the  $t$ - $T$  search. However,  
418 imposing constraints where the model is less sensitive leads to exclusion of (potentially viable) solutions and  
419 therefore tightens the envelope of accepted  $t$ - $T$  paths. These results suggest that data quantity, quality, the use of  $t$ - $T$   
420 constraint boxes, and proper data interpretation variably trade-off with one another. Figure 3E shows the ideal case  
421 with the most accurate thermal history recovery (nearly identical to the true history) when two constraint boxes are  
422 implemented with three interpreted AFT kinetic populations and three AHe grains modelled using the proper  
423 kinetics. Importantly, this applies in the case of integrating multiple low-temperature thermochronometers and/or  
424 multikinetic AFT data, especially multikinetic populations that progressively diverge in kinetics, therefore  
425 increasing thermal resolution. However, constraint boxes provide no obvious advantage when the three multikinetic  
426 populations are ignored and only the overall central AFT age is modelled (fig. 3H).



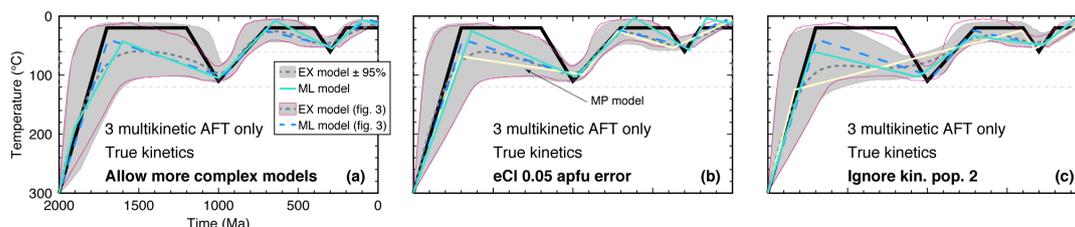
427

428 We show additional QTQt models in figure 5 to further demonstrate that multikinetic AFT data alone resolve the  
429 true history in the absence of explicit constraint boxes, due to enhanced temperature sensitivity from grain  
430 populations with distinct AFT kinetics. These simulations were carried out to test how robust the inference was for  
431 the “AFT only” models in figure 3 and we show the MP model here for comparison in this case when we are further  
432 exploring resolving power. We ran inversions where more complex models were allowed (fig. 5A; i.e., t–T points  
433 were added even if they did not provide better fits to the observed data), where additional noise was added in the  
434 form of uncertainty on the kinetic parameter value (fig. 5B), and lastly, we ignored the second AFT kinetic  
435 population during inversion (fig. 5C). All other conditions were the same as in previous model runs. At face value, it  
436 seems counter intuitive that we can resolve pre-thermal-maximum temperatures during reheating events because of  
437 how fission-track lengths respond to heating, therefore our resolution should disappear if more complex models are  
438 allowed.

439

440 Figure 5A illustrates that in this case there is some resolution lost for the EX model envelope when more complex  
441 models are allowed, in comparison to the EX path envelope in figure 3A. The figure 5A ML path is very similar to  
442 that in figure 3A and the MP path is identical to the ML model. Adding noise to the kinetic data ( $\pm 0.05$  apfu)  
443 creates little difference between the EX envelopes for the two models for the late history (fig. 5B), signifying a well-  
444 resolved solution ensemble, yet decreased ability to determine the timing and maximum temperature of the second  
445 reheating event and some loss of resolution in the pre-maximum-heating portion of the history. Yet overall there is  
446 not much difference between the figure 3A and figure 5B models, implying that some kinetic uncertainty is not  
447 critically detrimental (under the assumption that our kinetic models are completely accurate, which we know they  
448 are not). The most impactful choice affecting the multikinetic “AFT only” simulations was ignoring kinetic  
449 population two (fig. 5C). The loss of the ~840 Ma fluorapatite AFT population degraded our overall EX envelope t–  
450 T resolution near the peak temperature and affected the ability to recover the timing of the peak temperature. This is  
451 a reasonable consequence because the ~840 Ma AFT age is set upon cooling after the thermal maximum, pinning  
452 the timing and magnitude of heating. The ML and MP models are considerably different and reflect the loss of  
453 independent constraining information within the overall solution pool and demonstrate that without this kinetic  
454 group the “simple” model path is nearly continuous cooling and still adequately explains the observed data.  
455 Accepting more complex models and adding noise led to the loss of t–T resolution, suggesting that our conclusions  
456 are somewhat conditional on the assumptions of preferring model simplicity if there is low signal/noise or fewer  
457 constraining input data. The figure 5C simulation supports the argument that the addition of independent t–T  
458 information (i.e., multiple thermochronometers or AFT kinetic populations) enhances thermal history recovery  
459 without the requirement of constraint boxes.

460



461

462 **Figure 5:** Multikinetic AFT-only QTQt models without constraint boxes and same inversion setup as figure 3A–C. (A) QTQt run  
463 where more complex models were allowed. (B) Inversion where noise was added in the form of  $\pm 0.05$  apfu to the kinetic  
464 parameter. (C) Inversion where kinetic population two was ignored. All models: Magenta outline and long dashed blue line are  
465 the respective EX model 95% credible interval and ML model path from figure 3A. Thick black line is the “true” thermal history  
466 from figure 1; coloured solid cyan and light yellow lines are the respective Maximum Likelihood (best fit) and Maximum  
467 Posterior model t–T paths from QTQt for these inversions, short dashed gray lines are the Expected model t–T path with light  
468 gray 95% credible interval envelope.

### 469 5.3 The use of constraint boxes in t–T modelling

470 The addition of constraint boxes for models with low t–T resolution may yield a false sense of precision in some  
471 cases and suggests to us that boxes in QTQt should be used with caution when: (1) thermochronometer sensitivity is  
472 marginal or only one chronometer is used, (2) history complexity is presumed to be high, and (3) when histories  
473 approach  $10^8$ – $10^9$  timescales. The use of excessively tiny t–T boxes in QTQt may cause an unintentional linearising  
474 bias or artifact to occur because of the Bayesian treatment of user constraints for the prior probability during  
475 modelling. Essentially, this means t–T paths may be extremely linear between boxes if more complex models are  
476 prohibited. There is also the fact that paths are *required* to pass through a given t–T box. This is especially  
477 problematic for geologic histories involving unconformities where, for example, we know basement rocks were at  
478 surface by 450 Ma because there are preserved sediments of that age nearby. However, this information does not  
479 preclude samples actually being exhumed close to the surface at 650 Ma and sitting at or near the surface for 200  
480 million years before the deposition of Ordovician sediments. During Bayesian modelling, undue influence on the t–  
481 T search may occur if a constraint box were implemented at  $450 \pm 10$  Ma (depositional age) and  $10 \pm 10$  °C (surface  
482 temperature). We may suspect an issue if the majority of thermal histories showed a very linear, preferred t–T  
483 segment through our constraint box, yet some more complicated histories with a greater number of t–T points (i.e.,  
484 penalized more complex paths) were also visible yet exhibited cooling prior to our box constraint, albeit with less  
485 frequency. Unfortunately, under random Monte Carlo modelling assumptions, this “box biasing” would never be  
486 recognized as a problem due to the reliance on boxes for informing and expediting the t–T search. It is nonetheless  
487 difficult to generalize the use of constraint boxes for inverse modelling and outcomes ultimately depend on how  
488 constraints are implemented. Consequently, it is important to simulate and report several scenarios using different  
489 explicit conditions for the t–T search for complex histories (e.g., Gallagher and Ketcham, 2018).

490

491 When confronted with using one or two low-temperature thermochronometers over longer timescales, the choice  
492 that is often made is to add more t–T boxes to better delineate the model space. However, this opens the door for  
493 assumptions to be heralded as geologic evidence, and as we see in these examples, this can still yield inappropriate  
494 thermal histories when t–T resolution is low. It is important to differentiate between geologic constraints (e.g.,



495 stratigraphic relationship or basement nonconformity) and assumptions (e.g., regionally rocks cooled below  
496  $^{40}\text{Ar}/^{39}\text{Ar}$  biotite closure temperature of  $\sim 300$  °C at  $\sim 2000$  Ma) and to test different scenarios during modelling.  
497 There are obviously exceptions to these points but this simply means that a model outcome is only as good as the  
498 input data, and that tackling a complex problem with high expectations and few data should not — and cannot —  
499 result in exceptional model results without numerous assumptions and choices made by the modeller. That being  
500 said, we disagree with the recent assertion by Green and Duddy (2020) that “*thermochronology data in isolation*  
501 *cannot define periods when samples were cooler and subsequently reheated. This can only be defined with the aid of*  
502 *constraints from geological evidence.*” This statement alludes to the non-uniqueness of t–T models and applies in  
503 situations where a single AFT age population is modelled, or more generally when only one thermochronometer is  
504 used to elucidate complicated t–T histories. However, we propose that multikinetic AFT interpretations (or more  
505 generally, integration of independent information from multiple chronometers) demonstrate that their view does not  
506 always apply, as we can see illustrated in figure 3A. Green and Duddy (2020) also go on to state that slow,  
507 continuous cooling is often assumed in published thermal history models and that this is inappropriate. Of course,  
508 ignoring geologic information and blindly inputting thermochronology data into modelling software will always  
509 yield inappropriate thermal histories — and there is nothing preventing the user from doing this. However, model  
510 simulations such as the one that we show in figure 3G tell us that the wrong model may *imply* slow monotonic  
511 cooling, although it is not outright assumed, whereas our examples that utilize high-quality data (fig. 3A–E)  
512 demonstrate that universal slow cooling suppositions are invalid.

#### 513 **5.4 Thermochronometer kinetics and future considerations**

514 Our modelling results reveal that attempts to understand or interpret latent multikinetic age populations within AFT  
515 data provide meaningful information that makes sense within the appropriate kinetic context of both the AFT and  
516 AHe thermochronometers. This demonstrates that excess age scatter for AHe dates may be governed solely by  
517 composition (i.e.,  $r_{\text{mr0}}$ ) for grains of the same morphology and U content — and in this case, individual AHe dates  
518 may be older than an AFT central age for valid reasons. These t–T models based on synthetic data reinforce the  
519 conclusions of Gautheron et al. (2013) that specifically examined the track annealing law with respect to apatite  
520 grain chemistry ( $r_{\text{mr0}}$ ) and the effects on AHe dates. Our results are even more telling when one considers that the  
521 elemental data required to calculate  $r_{\text{mr0}}$  are often not collected and the typical AHe fluorapatite ( $r_{\text{mr0}} = 0.83$ )  
522 assumption can be misrepresentative for t–T modelling and produce inaccurate thermal history results, which was  
523 recently demonstrated with natural samples by Recanati et al. (2017) and Powell et al. (2020). This implies that  
524 radiation damage effects on He diffusivity are only one piece of the “kinetic puzzle” and that serious t–T  
525 inaccuracies can propagate into thermal history modelling if apatite composition, and therefore kinetics, are  
526 incorrectly determined or simply assigned as some default value for both the AFT and AHe methods. The distorted  
527 thermal history results shown here for the commonly used RDAAM (assuming uniform  $r_{\text{mr0}}$ ) support the adoption  
528 and routine use of a He kinetic model independent of fission track annealing kinetics (Gerin et al., 2017; Willett et  
529 al., 2017). This is especially applicable to deep-time histories where rocks have greater potential to undergo cyclical  
530 burial and exhumation because there is a greater age discrepancy introduced by the RDAAM when rocks spend



531 extended time at temperatures  $< 80$  °C (Willett et al., 2017). Another conclusion is that, in the case of examples such  
532 as figure 4F (or fig. 4I), the poorly predicted AHe dates are a result of an incorrect kinetic assumption and have  
533 nothing to do with the quality of the dates. In many cases, in light of the known age scatter issue surrounding the  
534 AHe method, workers may suspect these dates are erroneous because t–T modelling is not exhibiting agreement  
535 between observed and model dates. We stress that this disagreement could also be a result of incorrect model or  
536 geologic assumptions and have less to do with the observed age data, which further emphasizes an incomplete  
537 understanding of kinetics in our models. The implications of our modelling exercises would also advise against  
538 attempts to arbitrarily cull AHe datasets of “outliers” and recommend this should be carried out only in extreme,  
539 obvious cases of internal date disagreement or where other evidence is brought to bear on the source of excess age  
540 scatter, such as analytical screening of apatite degassing behaviour (Idleman et al., 2018; McDannell et al., 2018).  
541 Some of the examples discussed in McDannell et al. (2018), such as the Sierra Nevada apatite suite, exhibit similar  
542 grain sizes, U content, and CRH degassing behaviour nearly identical to Durango apatite (the common laboratory  
543 age standard), yet intrasample age scatter persists. Those experiments established that there are still variables that  
544 affect He diffusion that are difficult to characterize, such as the siting of He in the crystal lattice, how radiogenic He  
545 diffuses or is trapped during the transition from an open-to-closed system, and the specifics of how He is liberated  
546 during laboratory heating. This persistent age scatter may be partly explained by differences in He retentivities (i.e.,  
547 grain chemistry) that were unable to be documented directly by degassing patterns alone in those experiments.

548

549 We advise against the growing practice of trying to smooth out He date dispersion by arbitrarily binning grains into  
550 groups by effective uranium (eU, parent U, Th, and Sm content weighted for  $\alpha$  productivity) that can be represented  
551 by an average date (cf., Anderson et al., 2017; Anderson et al., 2018; Weisberg et al., 2018). This presupposes that  
552 the age dispersion is caused by analytical uncertainty rather than geological factors such as variable composition and  
553 radiation damage, even when we know the latter to be the prevailing sources of scatter. Although binning may make  
554 it easier to fit the data using simple model assumptions, it can potentially lead to distorted, close-fitting thermal  
555 histories that are interpreted as good solutions. Such an approach can act as a disincentive to acquire the data needed  
556 to better understand the causes for the age dispersion. Ultimately more research is necessary to address this issue  
557 but, in the absence of definitive data, it is worth trying to determine the model parameters that are needed to fit the  
558 observations. The advantage of Bayesian models like QTQt is that kinetic parameters are acknowledged to be  
559 uncertain and attempts are made to accommodate complicated thermochronologic datasets by adjusting kinetic  
560 parameters within specified uncertainty ranges. This is preferable to rejecting datasets that are deemed incompatible  
561 because they cannot be reconciled using simple modelling approaches with invariant parameters.

562

563 Currently there are limited options for directly quantifying kinetics for AHe grains outside of traditional (U–Th)/He  
564 step-heating or CRH diffusion experiments. However, the maturation of in situ laser ablation (U–Th)/He (Boyce et  
565 al., 2006; Pickering et al., 2020) provides a non-destructive micro-analytical method (at the expense of diffusion  
566 information) that yields age information and may improve sample characterization by allowing auxiliary elemental  
567 analysis. A way to bridge the gap and provide insight into possible chemical heterogeneity of unknowns is to carry



568 out AFT analyses and have grain mounts characterized by electron microprobe to obtain apatite elemental data while  
569 these methods are further developed and adopted. This approach will give a first-order approximation of the spread  
570 in apatite chemistry for aliquots analyzed for (U–Th)/He (e.g., Powell et al., 2020). We envision coupling laser  
571 ablation inductively coupled plasma mass spectrometry (LA–ICP–MS) AFT and AHe double-dating as a  
572 methodology that will prove valuable for better characterization of apatite chemistry in relation to derived dates,  
573 which will inform and require future laboratory experiments to be conducted on a diverse suite of apatites to better  
574 constrain annealing behaviour and extrapolate it to geologic timescales.

575

576 We recommend the routine collection of elemental data for apatite dated using the fission track method as a means  
577 to better quantify sample chemical variation and relate this to kinetic behaviour for thermal history analysis. The use  
578 of  $r_{\text{mfo}}$  (or eCl), while imperfect, still provides the best resolution for kinetic interpretation. The use of other kinetic  
579 parameters such as  $D_{\text{par}}$  are generally inadequate for more high-level kinetic interpretation due to low precision  
580 (Issler et al., 2018; McDannell et al., 2019b; Schneider and Issler, 2019), whereas the sole use of measured Cl is  
581 only applicable for a limited suite of apatites (i.e., chlorapatite) and neglects the influence of other elemental  
582 substitutions on annealing (i.e., Carlson et al., 1999; Barbarand et al., 2003). These topics are discussed more fully  
583 in a future companion paper that examines detrital AFT samples from Yukon, Canada to illustrate multikinetic AFT  
584 interpretation and modelling methods.

## 585 6. Conclusions

586 Using synthetic data derived from forward modelling, we show that, under ideal conditions, it is possible to extract  
587 multi-cyclic heating and cooling history information from multikinetic AFT and AHe data using inverse modelling  
588 methods when kinetic parameters for AFT annealing and AHe diffusion are correctly specified. Essential details of a  
589 two-phase heating and cooling history are reproduced using AFT multikinetic data alone without imposing  
590 constraint boxes but the closest fit to the true solution is achieved using all the synthetic data with constraint boxes.  
591 Alternative monokinetic interpretations that ignore multikinetic behaviour generate solutions that significantly  
592 depart from the true solution while providing close fits to the reinterpreted AFT data; under these conditions,  
593 imposing constraint boxes can make the  $t$ – $T$  solutions worse. Recent publications suggest that composition can  
594 influence He diffusion in apatite; in the context of our simulations, ignoring composition causes misfits to AHe  
595 dates and degrades model thermal histories. These results suggest that apatite elemental data should be acquired for  
596 interpreting and modelling overdispersed thermochronological datasets that result from multikinetic AFT annealing  
597 and AHe diffusion behaviour. The ability to recover high-resolution thermal histories from natural multikinetic AFT  
598 samples depends on the details of the thermal history and characteristics of the data and this is the subject of a future  
599 paper.

## 600 7. Appendix

601 The appendix contains the true thermal history and the synthetic AFT data set. See the main text for further details.



602 **8. Author Contributions**

603 KTM designed research and performed modelling. DRI was involved in conceptual discussions, model evaluation,  
604 and editing and drafting the manuscript. KTM wrote the paper.

605 **9. Competing interests**

606 The authors declare that they have no conflict of interest.

607 **10. Acknowledgements**

608 The authors graciously thank Kerry Gallagher for making changes to QTQt specifically for these modelling  
609 exercises. KTM also thanks his partner Jennifer for all of her help during the writing of this manuscript.

610 **References**

611 Anderson, A. J., Hodges, K. V., and van Soest, M. C.: Empirical constraints on the effects of radiation damage on  
612 helium diffusion in zircon, *Geochimica et Cosmochimica Acta*, 218, 308-322, 2017.

613 Anderson, A. J., Hodges, K., and Van Soest, M.: Comment on 'Distinguishing slow cooling versus multiphase  
614 cooling and heating in zircon and apatite (U-Th)/He datasets: The case of the McClure Mountain syenite standard'  
615 by Weisberg, Metcalf, and Flowers, *Chemical Geology*, 498, 150-152,  
616 <https://doi.org/10.1016/j.chemgeo.2018.07.006>, 2018.

617 Barbarand, J., Carter, A., Wood, I., and Hurford, T.: Compositional and structural control of fission-track annealing  
618 in apatite, *Chemical Geology*, 198, 107-137, [https://doi.org/10.1016/S0009-2541\(02\)00424-2](https://doi.org/10.1016/S0009-2541(02)00424-2), 2003.

619 Boyce, J., Hodges, K., Olszewski, W., Jercinovic, M., Carpenter, B., and Reiners, P. W.: Laser microprobe (U-  
620 Th)/He geochronology, *Geochimica et Cosmochimica Acta*, 70, 3031-3039,  
621 <https://doi.org/10.1016/j.gca.2006.03.019>, 2006.

622 Burtner, R. L., Nigrini, A., and Donelick, R. A.: Thermochronology of Lower Cretaceous source rocks in the Idaho-  
623 Wyoming thrust belt, *AAPG Bulletin*, 78, 1613-1636, <https://doi.org/10.1306/A25FF233-171B-11D7-8645000102C1865D>, 1994.

625  
626 Carlson, W. D.: Mechanisms and kinetics of apatite fission-track annealing, *American mineralogist*, 75, 1120-1139,  
627 1990.

628 Carlson, W. D., Donelick, R. A., and Ketcham, R. A.: Variability of apatite fission-track annealing kinetics: I.  
629 Experimental results, *American mineralogist*, 84, 1213-1223, 1999.

630 Carpena, J., Kienast, J.-R., Ouzegane, K., and Jehanno, C.: Evidence of the contrasted fission-track clock behavior  
631 of the apatites from In Ouzzal carbonatites (northwest Hoggar): The low-temperature thermal history of an Archean  
632 basement, *Geological Society of America Bulletin*, 100, 1237-1243, 1988.

633 Carpena, J. and Lacout, J.-L.: Thermal annealing of fission tracks in synthetic apatites, *Nuclear Instruments and  
634 Methods in Physics Research Section B: Beam Interactions with Materials and Atoms*, 268, 3191-3194, 2010.

635 Crowley, K., Cameron, M., and McPherson, B.: Annealing of etchable fission-track damage in F-, OH-, Cl- and Sr-  
636 apatite: 1. Systematics and preliminary interpretations, *International Journal of Radiation Applications and  
637 Instrumentation. Part D. Nuclear Tracks and Radiation Measurements*, 17, 409-410, 1990.



- 638 Danišik, M.: Integration of fission-track thermochronology with other geochronologic methods on single crystals.  
639 In: *Fission-Track Thermochronology and its Application to Geology*, Malusa, M. G. and Fitzgerald, P. (Eds.),  
640 Springer International Publishing, New York, 2019.
- 641 Djimbi, D. M., Gautheron, C., Roques, J., Tassan-Got, L., Gerin, C., and Simoni, E.: Impact of apatite chemical  
642 composition on (U-Th)/He thermochronometry: An atomistic point of view, *Geochimica et Cosmochimica Acta*,  
643 167, 162-176, 2015.
- 644 Donelick, R. A.: A method of fission track analysis utilizing bulk chemical etching of apatite. In: U.S. Patent  
645 5,267,274, United States, 1993.
- 646 Ehlers, T. A. and Farley, K. A.: Apatite (U-Th)/He thermochronometry: methods and applications to problems in  
647 tectonic and surface processes, *Earth and Planetary Science Letters*, 206, 1-14, 2003.
- 648 Farley, K., Blythe, A., and Wolf, R.: Apatite helium ages: comparison with fission track ages and track-length-  
649 derived thermal models, *EOS*, 277, F644, 1996.
- 650 Farley, K. A.: Helium diffusion from apatite; general behavior as illustrated by Durango fluorapatite, *Journal of*  
651 *Geophysical Research*, 105, 2903-2914, 2000.
- 652 Fitzgerald, P. G., Baldwin, S. L., Webb, L. E., and O'Sullivan, P. B.: Interpretation of (U-Th)/He single grain ages  
653 from slowly cooled crustal terranes: A case study from the Transantarctic Mountains of southern Victoria Land,  
654 *Chemical Geology*, 225, 91-120, 2006.
- 655 Flowers, R. M. and Kelley, S. A.: Interpreting data dispersion and “inverted” dates in apatite (U-Th)/He and fission-  
656 track datasets: An example from the US midcontinent, *Geochimica et Cosmochimica Acta*, 75, 5169-5186, 2011.
- 657 Flowers, R. M., Ketcham, R. A., Shuster, D. L., and Farley, K. A.: Apatite (U-Th)/He thermochronometry using a  
658 radiation damage accumulation and annealing model, *Geochimica et Cosmochimica Acta*, 73, 2347-2365, 2009.
- 659 Fox, M. and Shuster, D. L.: The influence of burial heating on the (U-Th)/He system in apatite: Grand Canyon case  
660 study, *Earth and Planetary Science Letters*, 397, 174-183, 2014.
- 661 Galbraith, R. and Green, P.: Estimating the component ages in a finite mixture, *International Journal of Radiation*  
662 *Applications and Instrumentation. Part D. Nuclear Tracks and Radiation Measurements*, 17, 197-206, 1990.
- 663 Galbraith, R. F. and Laslett, G. M.: Statistical models for mixed fission track ages, *Nuclear Tracks and Radiation*  
664 *Measurements*, 21, 459-470, 1993.
- 665 Gallagher, K.: Transdimensional inverse thermal history modeling for quantitative thermochronology, *Journal of*  
666 *Geophysical Research: Solid Earth*, 117, 1-16, 2012.
- 667 Gallagher, K. and Ketcham, R. A.: Comment on “Thermal history modelling: HeFTy vs. QTQt” by Vermeesch and  
668 Tian, *Earth-Science Reviews* (2014), 139, 279–290, *Earth Science Reviews*, 176, 387-394, 2018.
- 669 Gallagher, K. and Ketcham, R. A.: Comment on the reply to the Comment on “Thermal history modelling: HeFTy  
670 vs. QTQt” by Vermeesch and Tian, *Earth-Science Reviews* (2014), 139, 279–290, *Earth-Science Reviews*, 203,  
671 2020.
- 672 Gautheron, C., Barbarand, J., Ketcham, R. A., Tassan-Got, L., van der Beek, P., Pagel, M., Pinna-Jamme, R.,  
673 Couffignal, F., and Fialin, M.: Chemical influence on  $\alpha$ -recoil damage annealing in apatite: Implications for (U-  
674 Th)/He dating, *Chemical Geology*, 351, 257-267, 2013.
- 675 Gautheron, C., Tassan-Got, L., Barbarand, J., and Pagel, M.: Effect of alpha-damage annealing on apatite (U-  
676 Th)/He thermochronology, *Chemical Geology*, 266, 157-170, 2009.



- 677 Gerin, C., Gautheron, C., Oliviero, E., Bachelet, C., Djimbi, D. M., Seydoux-Guillaume, A.-M., Tassan-Got, L.,  
678 Sarda, P., Roques, J., and Garrido, F.: Influence of vacancy damage on He diffusion in apatite, investigated at  
679 atomic to mineralogical scales, *Geochimica et Cosmochimica Acta*, 197, 87-103, 2017.
- 680 Gleadow, A. and Duddy, I.: A natural long-term track annealing experiment for apatite, *Nuclear Tracks*, 5, 169-174,  
681 1981.
- 682 Glotzbach, C., Van Der Beek, P. A., and Spiegel, C.: Episodic exhumation and relief growth in the Mont Blanc  
683 massif, Western Alps from numerical modelling of thermochronology data, *Earth and Planetary Science Letters*,  
684 304, 417-430, 2011.
- 685 Green, P. and Duddy, I.: Discussion: Extracting thermal history from low temperature thermochronology. A  
686 comment on recent exchanges between Vermeesch and Tian and Gallagher and Ketcham, *Earth Science Reviews*,  
687 doi: <https://doi.org/10.1016/j.earscirev.2020.103197>, 2020.
- 688 Green, P., Duddy, I., Gleadow, A., Tingate, P., and Laslett, G.: Fission-track annealing in apatite: track length  
689 measurements and the form of the Arrhenius plot, *Nuclear Tracks and Radiation Measurements*, 10, 323-328, 1985.
- 690 Green, P., Duddy, I., Gleadow, A., Tingate, P., and Laslett, G.: Thermal annealing of fission tracks in apatite: 1. A  
691 qualitative description, *Chemical Geology: Isotope Geoscience section*, 59, 237-253, 1986.
- 692 Green, P. F., Crowhurst, P. V., Duddy, I. R., Japsen, P., and Holford, S. P.: Conflicting (U-Th)/He and fission track  
693 ages in apatite: enhanced He retention, not anomalous annealing behaviour, *Earth and Planetary Science Letters*,  
694 250, 407-427, 2006.
- 695 Guenther, W. R., Reiners, P. W., Ketcham, R. A., Nasdala, L., and Giester, G.: Helium diffusion in natural zircon:  
696 Radiation damage, anisotropy, and the interpretation of zircon (U-Th)/He thermochronology, *American Journal of  
697 Science*, 313, 145-198, 2013.
- 698 Hendriks, B. and Redfield, T.: Apatite fission track and (U-Th)/He data from Fennoscandia: An example of  
699 underestimation of fission track annealing in apatite, *Earth and Planetary Science Letters*, 236, 443-458, 2005.
- 700 House, M. A., Wernicke, B. P., and Farley, K. A.: Dating topography of the Sierra Nevada, California, using apatite  
701 (U-Th)/He ages, *Nature*, 396, 66-69, 1998.
- 702 Idleman, B. D., Zeitler, P. K., and McDannell, K. T.: Characterization of helium release from apatite by continuous  
703 ramped heating, *Chemical Geology*, 476, 223-232, 2018.
- 704 Issler, D. R., Lane, L. S., and O'Sullivan, P. B.: Characterisation, interpretation and modelling of multi-kinetic  
705 apatite fission track data using elemental data. Geological Survey of Canada, Scientific Presentation 94, 1 sheet,  
706 <https://doi.org/10.4095/311302>, 2018.
- 707 Ketcham, R. A.: Fission-track annealing: From geologic observations to thermal history modeling. In: *Fission-Track  
708 Thermochronology and its Application to Geology*, Springer, 2019.
- 709 Ketcham, R. A., Carter, A., Donelick, R. A., Barbarand, J., and Hurford, A. J.: Improved modeling of fission-track  
710 annealing in apatite, *American Mineralogist*, 92, 799-810, 2007.
- 711 Ketcham, R. A., Donelick, R. A., and Carlson, W. D.: Variability of apatite fission-track annealing kinetics; III,  
712 Extrapolation to geological time scales, *American Mineralogist*, 84, 1235-1255, 1999.
- 713 Ketcham, R. A., Gautheron, C., Recanati, A., and Rahn, M.: Possible influence of alpha recoil track percolation on  
714 helium diffusivity in apatite, *Goldschmidt Abstracts*, Paris, France, 1983, 2017.



- 715 Kohn, B. P., Lorencak, M., Gleadow, A. J., Kohlmann, F., Raza, A., Osadetz, K. G., and Sorjonen-Ward, P.: A  
716 reappraisal of low-temperature thermochronology of the eastern Fennoscandia Shield and radiation-enhanced apatite  
717 fission-track annealing, Geological Society, London, Special Publications, 324, 193-216, 2009.
- 718 McDannell, K. T. and Flowers, R. M.: Vestiges of the Ancient: Deep-Time Noble Gas Thermochronology,  
719 Elements, 16, 325-330, 2020.
- 720 McDannell, K. T., Issler, D. R., and O'Sullivan, P. B.: Radiation-enhanced fission track annealing revisited and  
721 consequences for apatite thermochronometry, Geochimica et Cosmochimica Acta, 252, 213-239, 2019a.
- 722 McDannell, K. T., Schneider, D. A., Zeitler, P. K., O'Sullivan, P. B., and Issler, D. R.: Reconstructing deep-time  
723 histories from integrated thermochronology: An example from southern Baffin Island, Canada, Terra Nova, 31, 189-  
724 204, 2019b.
- 725 McDannell, K. T., Zeitler, P. K., Janes, D. G., Idleman, B. D., and Fayon, A. K.: Screening apatites for (U-Th)/He  
726 thermochronometry via continuous ramped heating: He age components and implications for age dispersion,  
727 Geochimica et Cosmochimica Acta, 223, 90-106, 2018.
- 728 Naeser, N. D., Naeser, C. W., and McCulloch, T. H.: The application of fission-track dating to the depositional and  
729 thermal history of rocks in sedimentary basins. In: Thermal History of Sedimentary Basins, Naeser, N. D. and  
730 McCulloch, M. T. (Eds.), Springer-Verlag, 1989.
- 731 Pickering, J., Matthews, W., Enkelmann, E., Guest, B., Sykes, C., and Koblinger, B. M.: Laser ablation (U-Th-  
732 Sm)/He dating of detrital apatite, Chemical Geology, 548, <https://doi.org/10.1016/j.chemgeo.2020.119683>, 2020.
- 733 Powell, J. W., Issler, D. R., Schneider, D. A., Fallas, K. M., and Stockli, D. F.: Thermal history of the Mackenzie  
734 Plain, Northwest Territories, Canada: Insights from low-temperature thermochronology of the Devonian Imperial  
735 Formation, Geological Society of America Bulletin, 132, 767-783, 2020.
- 736 Ravenhurst, C. E., Roden, M. K., Willett, S. D., and Miller, D. S.: Dependence of fission track annealing kinetics on  
737 apatite crystal chemistry, Nuclear Tracks and Radiation Measurements, 21, 622, 1993.
- 738 Recanati, A., Gautheron, C., Barbarand, J., Missenard, Y., Pinna-Jamme, R., Tassan-Got, L., Carter, A., Douville,  
739 É., Bordier, L., and Pagel, M.: Helium trapping in apatite damage: Insights from (U-Th-Sm)/He dating of different  
740 granitoid lithologies, Chemical Geology, 470, 116-131, 2017.
- 741 Schneider, D. A. and Issler, D. R.: Application of low-temperature thermochronology to hydrocarbon exploration.  
742 In: Fission-Track Thermochronology and its Application to Geology, Malusa, M. G. and Fitzgerald, P. (Eds.),  
743 Springer International Publishing, New York, 2019.
- 744 Shuster, D. L. and Farley, K. A.: The influence of artificial radiation damage and thermal annealing on helium  
745 diffusion kinetics in apatite, Geochimica et Cosmochimica Acta, 73, 183-196, 2009.
- 746 Shuster, D. L., Flowers, R. M., and Farley, K. A.: The influence of natural radiation damage on helium diffusion  
747 kinetics in apatite, Earth and Planetary Science Letters, 249, 148-161, 2006.
- 748 Tello, C. A., Palissari, R., Hadler, J. C., Iunes, P. J., Guedes, S., Curvo, E. A., and Paulo, S. R.: Annealing  
749 experiments on induced fission tracks in apatite: Measurements of horizontal-confined track lengths and track  
750 densities in basal sections and randomly oriented grains, American mineralogist, 91, 252-260, 2006.
- 751 van der Beek, P., Andriessen, P., and Cloetingh, S.: Morphotectonic evolution of rifted continental margins:  
752 Inferences from a coupled tectonic-surface processes model and fission track thermochronology, Tectonics, 14, 406-  
753 421, 1995.
- 754 Vermeesch, P.: IsoplotR: A free and open toolbox for geochronology, Geoscience Frontiers, 9, 1479-1493, 2018.



- 755 Vermeesch, P. and Tian, Y.: Thermal history modelling: HeFTy vs. QTQt, *Earth-Science Reviews*, 139, 279-290,  
756 2014.
- 757 Weisberg, W. R., Metcalf, J. R., and Flowers, R. M.: Distinguishing slow cooling versus multiphase cooling and  
758 heating in zircon and apatite (U-Th)/He datasets: the case of the McClure Mountain syenite standard, *Chemical*  
759 *Geology*, 485, 90-99, 2018.
- 760 Willett, C. D., Fox, M., and Shuster, D. L.: A helium-based model for the effects of radiation damage annealing on  
761 helium diffusion kinetics in apatite, *Earth and Planetary Science Letters*, 477, 195-204, 2017.
- 762 Wolf, R. A., Farley, K. A., and Kass, D. M.: Modeling of the temperature sensitivity of the apatite (U-Th)/He  
763 thermochronometer, *Chemical Geology*, 148, 105-114, 1998.
- 764 Zeitler, P. K., Johnson, N. M., Naeser, C. W., and Tahirkheli, R. A.: Fission-track evidence for Quaternary uplift of  
765 the Nanga Parbat region, Pakistan, *Nature*, 298, 255-257, 1982.
- 766

767

## 768 Figure Captions

769

770 **Figure 1:** Thermal history used to predict synthetic AFT and AHe data. This t–T path is referred to as the “true” thermal history  
771 throughout this paper. The predicted synthetic data were then used as input for QTQt to recover the thermal history through  
772 inverse modelling. PAZ = partial annealing zone for fission tracks.

773 **Figure 2:** Predicted synthetic AFT data from the thermal history in figure 1. Multikinetik age populations were individually  
774 predicted using distinct  $r_{m0}$  kinetics shown in (B) panels (discussed in the text). These data were then input in QTQt and inverted  
775 in an attempt to recover the true thermal history in figure 1 (see fig. 3). (A) Central age and  $1\sigma$  errors are indicated for each  
776 kinetic population. Kinetic populations one, two, and three are displayed as arms on their respective radial plots, with individual  
777 AFT ages closer to the origin being less precise. The last radial plot shows all thirty individual grains and demonstrates that when  
778 taken together, the combined sample fails the  $\chi^2$  test ( $p < 0.05$ ) for homogeneity (i.e., that all grains belong to a single underlying  
779 age population) suggesting multiple age populations. This is the scenario most researchers would start with before evaluating the  
780 sample for potential multikinetik behaviour. Mixture modelling was subsequently performed on the combined sample and the  
781 model age peaks that were picked seamlessly align with the individual kinetic population central ages. This aligns with how  
782 populations would be defined and compared with the elemental chemistry for individual age grains during multikinetik  
783 interpretation. (B) The predicted track length distributions for each kinetic population from the thermal history in Figure 1 using  
784 the specified kinetic parameter value. The last panel on the right combines all tracks from each kinetic population. Numbers on  
785 the histogram are the number of tracks in each  $\mu\text{m}$  bin. Abbreviations: eCI = effective CI; MTL = mean track length.

786 **Figure 3:** Thermal history inversion results from QTQt under different imposed kinetic and t–T assumptions. (A–C) show the  
787 “AFT only” models that utilized three multikinetik AFT populations (discussed in the text) as the only input data. The true  $r_{m0}$   
788 kinetics applied during forward modelling were entered in the input files and held fixed for each kinetic population during the  
789 inversion. (D–E) show the results of models that correctly utilized three multikinetik AFT kinetic populations and three AHe  
790 dates all with the true kinetics held fixed. Panel E is the best model inversion incorporating all correct thermochronometer  
791 information used during forward modelling of the synthetic data set. The panel (F) model was completed under the same  
792 conditions as panels (D–E) except that the three AHe grains all employ the incorrect (in the oldest and youngest cases) RDAAM  
793 default fluorapatite  $r_{m0}$  value of 0.83 as the kinetic parameter. Panels (G–I) were modelled assuming a “monokinetik” or  
794 traditional single population AFT sample that combines all three multikinetik populations into one. For all panels: Thick black  
795 line is the “true” thermal history from figure 1; coloured, solid lines are the Maximum Likelihood model (best fit) t–T path from  
796 QTQt; dashed gray lines are the Expected model t–T path with light gray 95% credible interval envelope. Assumed t–T  
797 constraints are black boxes that require thermal histories to pass through them during the inversion.

798 **Figure 4:** QTQt inversion predictions compared to “observed” synthetic thermochronology data generated during forward  
799 modelling. Panel letters correspond to counterpart t–T model panels in figure 3. All predictions are for the Maximum Likelihood  
800 models. Squares are observed AFT central age  $\pm 2\sigma$ , circles are predicted AFT age, diamonds are observed MTL  $\pm 1\sigma$ , and X-  
801 symbols are the predicted MTL. Individual model fits to each track length distribution for the AFT kinetic populations are also  
802 shown and color-coded the same as figure 2. Observed apatite He dates shown by red H-symbol (spans the  $1\sigma$  error range quoted  
803 in the text) and predicted AHe dates are black bars. Panel E with star is our best model that accounts for all multikinetik AFT



804 populations and utilizes the true AHe kinetics and two geologic constraints, all combined for the highest thermal history  
805 resolution. Note: track length distributions are arbitrarily placed next to their respective age population and were not plotted with  
806 respect to the MTL plot axis.

807 **Figure 5:** Multikinetic AFT-only QTQt models without constraint boxes and same inversion setup as figure 3A–C. (A) QTQt run  
808 where more complex models were allowed. (B) Inversion where noise was added in the form of  $\pm 0.05$  apfu to the kinetic  
809 parameter. (C) Inversion where kinetic population two was ignored. All models: Magenta outline and long dashed blue line are  
810 the respective EX model 95% credible interval and ML model path from figure 3A. Thick black line is the “true” thermal history  
811 from figure 1; coloured solid cyan and light yellow lines are the respective Maximum Likelihood (best fit) and Maximum  
812 Posterior model  $t$ – $T$  paths from QTQt for these inversions, short dashed gray lines are the Expected model  $t$ – $T$  path with light  
813 gray 95% credible interval envelope