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Dear Dr. Wang and Dr. Oskin,

I apologise once again for the lengthy review process. It has not been easy to find qualified reviewers for your manuscript. However, Dr. Greg Balco kindly stepped in and provided a detailed and constructive review of your paper that largely agrees with my own opinion.

You have done a good job at addressing the comments on the first version of your paper, with one exception. The issue of negative inheritance, which I had raised in my own review of your original manuscript, is brushed aside and not dealt with in earnest.

I agree with Dr. Balco that this issue must be addressed in the final version of your paper, not only because it affects your uncertainty estimates, but also because it has wider implications for cosmogenic nuclide geochronology in general.

The reviewer succinctly summarises three inverse modelling approaches using two compact sets of equations. He argues that the second set of equations ('Problem 2') provides more meaningful uncertainty estimates than the unconstrained approach ('Problem 1') proposed in your paper. Dr. Balco's 'Problem 2' formulation is similar to the suggestion that I made in my review of your first submission:

> "A pragmatic way to fix this issue is to parameterise your problem in terms of the logarithm of inheritence, rather than inheritence itself. After the optimisation, you can then exponentiate the maximum likelihood value to ensure a strictly positive result."

Using similar notation to that used in Dr. Balco's review, the latter approach can be implemented by defining two new variables γ and τ so that $C_{inh} = e^\gamma$ and $T_e = e^\tau$. Then:

$$\text{given } X = (\gamma, \tau) \tag{3a}$$

$$\text{minimise } f(X) = \sum_i \left[(e^\gamma + P_i e^\tau) - C_i \right]^2 \tag{3b}$$

$$\text{over } X \text{ such that } -\infty < \gamma < \infty; -\infty < \tau < \infty. \tag{3c}$$

The resulting uncertainty distributions for C_{inh} and τ will be skewed (as in Dr. Balco's example), but will (probably) be unimodal rather than bimodal.

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Regardless of which approach you prefer, it is important that you discuss these options in the next iteration of your manuscript. You should also address the other points raised by Greg Balco. I will then make a final decision without requiring another round of review.

Sincerely yours,

Pieter Vermeesch