Response to review by Dr. Donald Davis

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We thank Dr. Davis for his positive and open minded review. We are happy that, after reading our manuscript, he has come to the conclusion that conventional isochrons should be avoided completely!

The reviewer is correct that the approximate symbol for the correlation coefficient in equation 4 derives from the fact that it is the first order approximation to a Taylor expansion. However, this is not the reason why the conventional and inverse isochron age estimates may disagree in the presence of large uncertainties. The actual reason is that isotopic ratios are strictly positive quantities with skewed error distributions. The weighted least squares algorithm of York (1969) does not take into account this skewness and this may cause the conventional and inverse isochron age to diverge.

To address this issue, we will replace the K–Ca example with a semi-synthetic version of it, to demonstrate that the inverse isochron is more accurate than the conventional isochron. The semi-synthetic dataset will be created as follows:

1. Let x_i be the i^{th} ^{40}K/^{44}Ca ratio (out of $n = 30$) from the Harrison et al. (2010) dataset, and let $\sigma[x_i]$, $\sigma[y_i]$, $\rho[x_i, y_i]$ be the standard errors and error correlation of the ^{40}K/^{44}Ca and ^{40}Ca/^{44}Ca ratios.

2. Collect n pairs of logratios $\{\ln[X_i], \ln[Y_i]\}$ from a bivariate normal distribution with means $\{\ln[x_i], \ln[y_i]\}$ and covariance matrix Σ_i where

$$y_i = y_\circ + 0.895 x_i (\exp[\lambda_{40} t] - 1) \quad (1)$$

   in which $y_\circ = 66$ is the initial ^{40}Ca/^{44}Ca ratio, $t = 800$ Ma is the true K–Ca age, and

$$\Sigma_i = \begin{bmatrix} \frac{1}{x_i} & 0 \\ 0 & \frac{1}{y_i} \end{bmatrix} \begin{bmatrix} \sigma[x_i]^2 & \rho[x_i, y_i]\sigma[x_i]\sigma[y_i] \\ \rho[x_i, y_i]\sigma[x_i]\sigma[y_i] & \sigma[x_i]^2 \end{bmatrix} \begin{bmatrix} \frac{1}{x_i} & 0 \\ 0 & \frac{1}{y_i} \end{bmatrix}$$

3. The semi-synthetic dataset is then given by $\{X_i, Y_i\}$ (for $1 \leq i \leq n$) with covariance matrices Σ_i' that are computed as follows:

$$\Sigma_i' = \begin{bmatrix} X_i & 0 \\ 0 & Y_i \end{bmatrix} \Sigma \begin{bmatrix} X_i & 0 \\ 0 & Y_i \end{bmatrix}$$

The logarithmic transformation is necessary to account for the inevitable skewness of the error distributions. Even though the semi-synthetic dataset is defined in terms of the conventional isochron equation (Eq. 1), it is the inverse isochron that most accurately estimates the age:

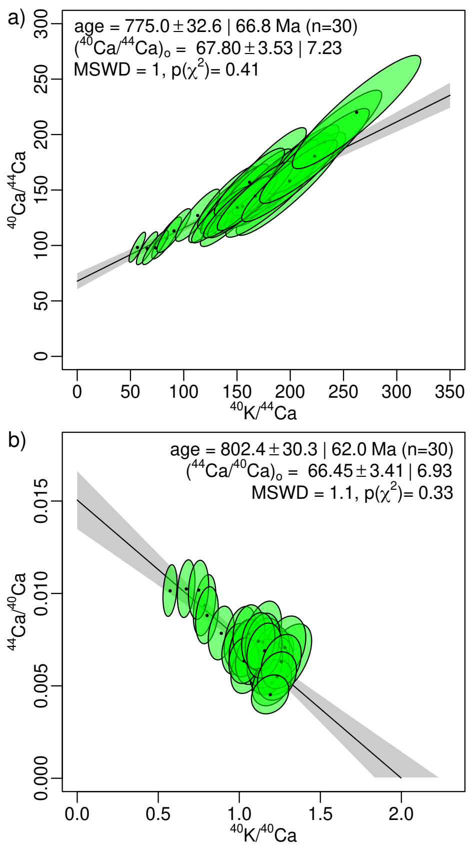

Figure 1: a) conventional and b) inverse isochron of a semi-synthetic dataset based on a K–Ca data of Harrison et al. (2010). The true age is 800 Ma and the true initial ^{40}Ca/^{44}Ca ratio is 66. The inverse isochron better approximates these values than the conventional isochron.

We have repeated this numerical experiment numerous times and the inverse isochron is always more accurate. A full theoretical discussion of this phenomenon falls outside the scope of our short communication. But we hope that this example satisfies the request from the reviewer, whom we would like to thank again for the useful suggestion.

References

Harrison, T. M., Heizler, M. T., McKeegan, K. D., and Schmitt, A. K.: In situ ^{40}K–^{40}Ca 'double-plus' SIMS dating resolves Klokken feldspar ^{40}K–^{40}Ar paradox, Earth and Planetary Science Letters, 299, 426–433, 2010.

York, D.: Least squares fitting of a straight line with correlated errors, Earth and Planetary Science Letters, 5, 320–324, 1969.