# Response to review by Dr. Ryan Ickert

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April 8, 2021

The detailed review by Dr. Ickert agrees with the contents of the paper, but is critical of certain aspects of the presentation.

> *Although the authors of this manuscript appear to be aware that their work is not new (line 66) they don't make clear what differentiates this contribution from others. The manuscript would be improved if it were better able to highlight a novel contribution.*

As the reviewer notes, we did not claim to have invented a new approach to geochronology. Our short communication simply aims to draw the attention of the wider geochronology community to the benefits of inverse isochrons. The positive review by Dr. Donald Davis proves that there is a need for such a paper. The Re–Os method will benefit from a switch to inverse isochrons, and so will the K–Ca and other methods. It is true that inverse ratios are widely used in Pb–Pb, U–Pb, Ar–Ar and Th–U geochronology. In each case, geochronologists have effectively 'reinvented the wheel'. By generalising inverse isochrons to all common chronometers in `IsoplotR`, our paper will hopefully represent the last time that such reinvention is necessary. Inverse isochrons have been part of `IsoplotR` for more than a year, yet nobody seems to have noticed this feature so far. Our short paper intends to change that.

Besides this modest goal of advertising a great graphical tool, our short paper makes two further 'novel' contributions:

1. As the reviewer points out, it provides a handy formula to convert conventional isotope ratios to inverse ratios. I am not aware of this calculation being documented in the geological literature.
2. In response to the two reviews, the revised version of the paper will demonstrate that the inverse isochron produces more accurate results than the conventional isochron.

The revised abstract will highlight these two aspects of the paper.

> *The argument that it more easily allows outlier identification is not particularly compelling: The Re-Os example in Fig 1C is unconvincing – the outliers they "identify" on the plot are not clear, at least*

*to me, and anyways that result is muddled somewhat by the fact that they have mixed samples of likely different ages on the same diagram (as described in the original paper). A better way to identify data that have undue weight on the MSWD is to simply inspect the variance normalized residuals and look for the largest values.*

The comment about the Morelli dataset being a mixture of three samples is well taken and will be acknowledged in the revised manuscript.

We attribute the dispersion to aliquots 1, 12 and 14 because their error ellipses have the least overlap with the isochron. This is how we understand most isochron users interpret their data. It may be so that inspecting the variance normalised residuals may be a better approach, but we have rarely seen this being used in practice.

*there is a persistent belief in some workers that ages determined by one regression are better or more precise than using an inverse or vice versa (e.g., Connelly et al., 2017). This would be trivial for the authors to include, by producing regression analysis on both the isochron and its inverse and demonstrating substantive equivalence. This is complicated somewhat by the fact that the two regressions become significantly distinct with extremely large uncertainties (as they state on line 100) and also with highly overdispersed data, but it is easy to carve out that as an exception.*

We will address this request by modifying Figures 1 and 2. The new versions of these figures will report the age and intercept for both the conventional and inverse isochron, thereby highlighting the differences between them. Figure 1 will become:

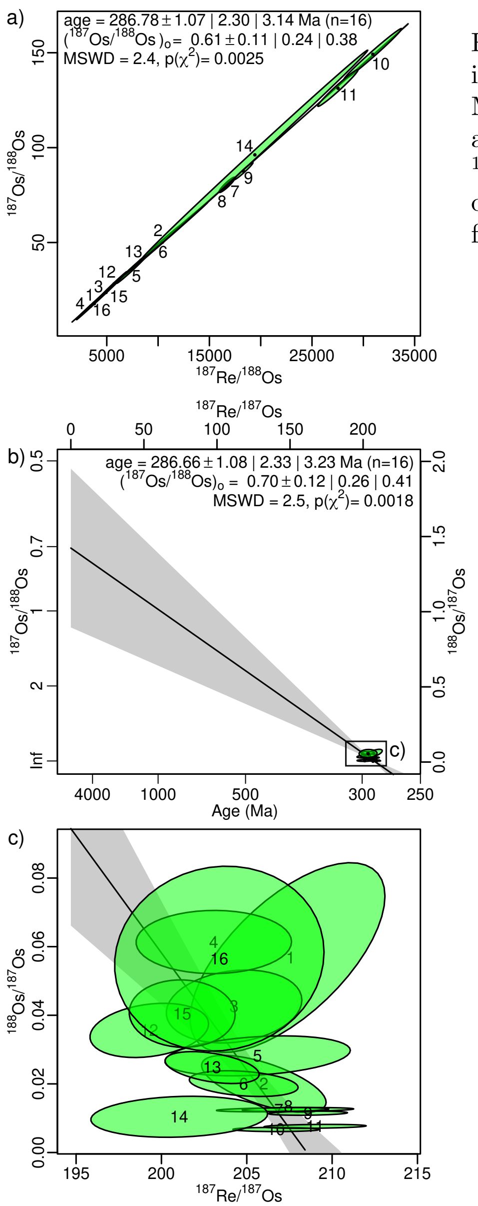


Figure 1: a) conventional and b), c) inverse isochron of the Re–Os data of Morelli et al. (2007). The isochron ages are similar but the inherited $^{187}$Os/$^{188}$Os-ratios are not. Note the double x- and y-axis of panel b), which follows a suggestion from Reviewer 1.

Figure 2 will be replaced by a semi-synthetic K–Ca dataset, which will be constructed as described in the response to Reviewer 1. This example demonstrates that, in the case of imprecise dataset, the inverse isochron produces more accurate results than the conventional isochron. We will also explore the effect of overdispersion without going into too much detail.

> *As written, the manuscript gives a misleading impression about the origin of correlations in isotopic and geochronological data. While*

*poor counting statistics on denominator isotopes may be important in some Ar isotope datasets, most uncertainty correlations in real, published datasets are due to other factors, such as fractionation corrections, interelement calibration, and blank corrections.*

This is definitely true for the Wetherill concordia diagram, in which the elemental fractionation between U and Pb is responsible for the error correlations. We will clarify this in the revised manuscript.

The 'Minor elements' in the second half of the review are all straightforward to address with the following exceptions:

*A spurious correlation is something more akin to the classic "pirates are causing global warming" example (and many others, cf. https://www.tylervigen.com/spurious-correlations). This word should not be used in the manuscript to describe any of the correlations, which are all real.*

In fact the word 'spurious' was meant exactly as intended by Karl Pearson (1897) in his classic paper "on a form of spurious correlation which may arise when indices are used in the measurement of organs", which is referenced in the paper. In the case of Re–Os geochronology, it is possible to observe a strong apparent correlation between the $^{187}$Os/$^{188}$Os and $^{187}$Re/$^{188}$Os ratio measurements when the correlation between the true atomic $^{187}$Os/$^{188}$Os and $^{187}$Re/$^{188}$Os ratios is in fact zero. We will add a sentence to clarify this source of apparent confusion.

*Section 4: This whole section seems superfluous. A statement at the end of the manuscript stating that "these calculations are implemented in Isoplot R" is sufficient. The paragraph and screen grab are unnecessary.*

We would like to point out that Figure 3 achieves a lot in a small amount of space:

1. It shows a second Re–Os example with weaker error correlations.
2. It shows how to perform the calculation from the command line, which may not be obvious for readers who are not familiar with `R`.
3. In the revised version of the manuscript, we will modify this figure so that it shows both the conventional and inverse isochron, which in this case produce essentially identical results due to the good precision and excellent spread of the data along the isochron.

*Line 99: What does "mathematically equivalent" mean?*

It means that, in the limit where $s[x_i]/x_i = 0$ and $s[y_i]/y_i = 0$, that the two formulations give exactly the same result. The reviewer uses the term "substantive equivalence" in his review and we would be happy to use the same term in the revised manuscript.

## References

Morelli, R., Creaser, R. A., Seltmann, R., Stuart, F. M., Selby, D., and Graupner, T.: Age and source constraints for the giant Muruntau gold deposit, Uzbekistan, from coupled Re-Os-He isotopes in arsenopyrite, Geology, 35, 795–798, 2007.