Answer to review by Green

A recipe for age dating thermal events and calculating temperature history is given before answering specifically, the comments by the 1. Reviewer. To begin with, we consider an idealized model with tracks generated continuously (not spontaneously), with no length spread during annealing, and no errors of measurements.

1. Construct a track length histogram of track counts of the randomly oriented tracks.
2. Convert the track length histogram into a histogram of time intervals by dividing the number of tracks by the rate of track generation. Each column of this histogram represents the time it takes to generate the number of tracks within the corresponding track length bin.
3. Cumulate the time intervals from the youngest to the oldest to achieve the track age distribution over the binned track lengths. A jump of the ages in this histogram dates a marked temperature change. The histogram of the pre-event tracks can now be constructed from the left part of the cumulated ages and the post-event histogram from the right part. Post-sedimentary tracks can be identified using the cumulated age distribution given the time of deposition. To this point, there has been no need for a track annealing law.
4. Calculate the temperature history based on the histogram of time intervals. Choose for example a piecewise linear function. Consider the rightmost column. The first age-temperature point is at age zero and with the present temperature. The next point back in time is at the age given by the left length boundary of the rightmost column of the histogram. Find the temperature at this point using an annealing law by adjusting the trial temperature until the track with the length at generation is annealed to the track length defining the leftmost part of the bin. The temperature is calculated in a few iteration steps because there is only one solution. This temperature is also the temperature at the right limit of the next bin. The temperature at the left limit of this bin is then calculated similarly. The temperature history is thus calculated backward in time as a stepwise linear function.

This recipe is based on the development by Bertagnolli (1983), Keil et al. (1987), and Jensen et al. (1992).

The recipe for the realistic case is presented next. Here confined horizontal etched tracks are considered together with the surface track density. Tracks are produced spontaneously, track lengths are spread during annealing, observed track lengths are biased, and measurements are with uncertainties.

1. Construct a track length histogram of the observed confined horizontal etched tracks.

1a. Remove the various obstacles that spread the annealed tracks over a greater length interval by deconvolving (de-blurring) the measured histogram. The new deconvolved histogram is then the basis for further treatment as in the recipe for the idealized tracks.

2. Convert the deconvolved track length histogram into a histogram of time intervals. The procedure is presented in the ms.
3. The temperature jump is age-dated as described above. The part of the measured histogram with tracks generated before the thermal event can now be identified by convolution of the left part of the deconvolved histogram. Convolution spreads the idealized tracks. For sedimentary samples, the post-sedimentary part of the measured histogram can now be identified. To this point, there has been no need for a track annealing law.
4. The temperature history is calculated from the deconvolved histogram as described for the idealized tracks.

The procedure for calculating the temperature history based on the deconvolved track length histogram was presented by Jensen et al. (1992). At that time, deconvolution was performed by trial and error. Later mathematical simulated annealing was introduced instead (Jensen and Hansen, 2018). The principle of age dating and identifying inherited tracks by the deconvolution is presented in the present ms. The deconvolution is here performed using Tarantola inversion. Calculations are direct with no use of Monte Carlo simulation.

Specific answers to the 1. reviewer

Wording:

I suggest a new title: Age dating thermal event by deconvolving confined horizontal fission track histograms. The wording ”Age distribution of fission tracks” is imprecise and should be changed to “Binned cumulated age distribution of the deconvolved track length histogram”. An example is shown in fig. 4c of the ms. Age is the age of the track generation.

L. 26-27. “independent of any annealing law”:

The annealing properties of the apatite mineral affect the final apparent age as well as the expected age of the oldest randomly oriented unetched track. However, the age of this track can be determined by counting the number of all tracks in a volume and divide by the track generation rate (eq. 1 in the ms). In the simplified case of no spread of tracks, the expected age of a given track is calculated by counting the number of shorter tracks and add 1. This age is determined without the use of any annealing law. The unetched tracks are not routinely measured and etched confined horizontal tracks are measured instead. The math in the ms explains how they can be used together with the surface track density instead of the unetched tracks.

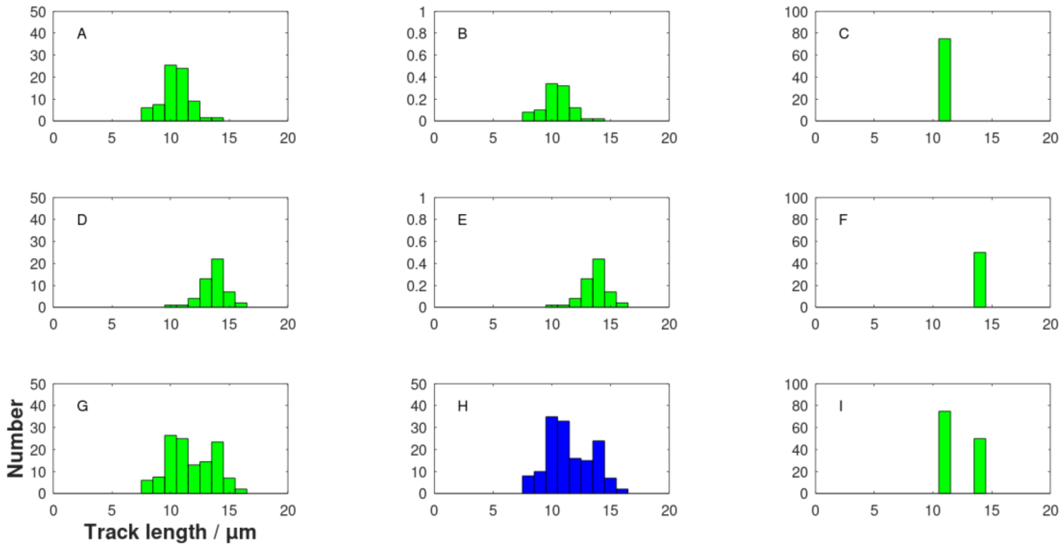
“pick apart”

After the formation of tracks, the spread of lengths increases during annealing leading to a considerable mixing of lengths of the combined track length histogram. There is not a one-to-one relationship between time and track length. However, the observed track length histogram is not blurred. There is still a tendency that the oldest tracks appear in the short track length part of the histogram and the newest tracks in the opposite part. De-blurring by deconvolution can to some extent reduce the spread. Deconvolution is used extensively in seismic processing where it increases the signal-to-noise ratio. This requires the character of the noise to be well known. The “noise” in connection with fission tracks is the spread observed in laboratory annealing experiments. This spread is used to reduce the spread of the observed track length histograms. The histogram can be de-blurred by deconvolution. The resulting deconvolved histogram consists of length ordered tracks. The age of the oldest track of each bin of this idealized histogram is determined by the counting principle with no use of an annealing law.

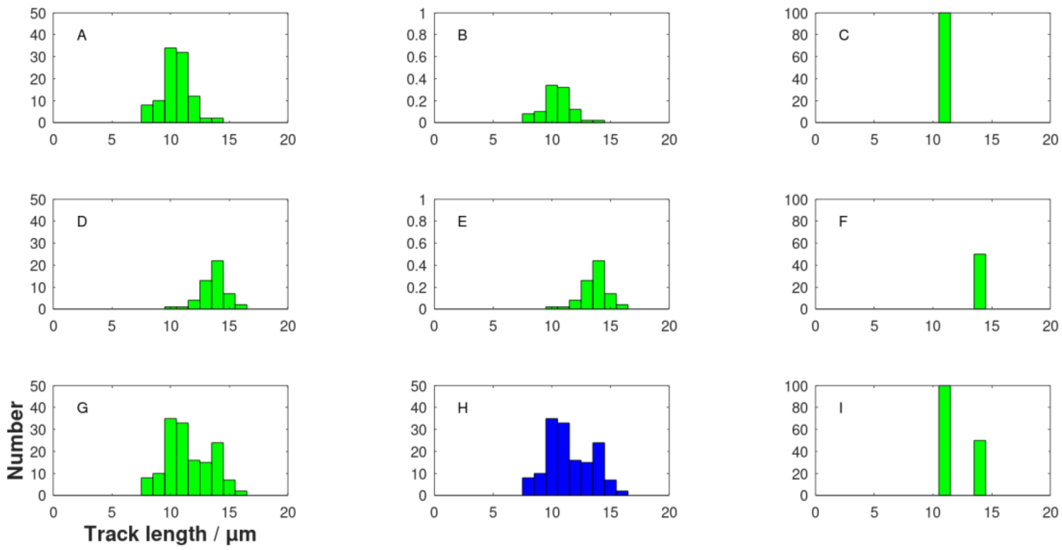
The 1. Reviewer presents an example of a thermal history leading to a bimodal track length histogram with an overlap between the two modes. Deconvolution can resolve the overlap and, in this way, determine the timing of the thermal events. There are limits to the resolution. If the two modes are too close to each other it is not possible. But, in the example given by the 1. Reviewer it is certainly possible because the overlap is not complete. The deconvolution procedure is illustrated in the Appendix below. The result of a “pick apart” of a two-mode 3D track length histogram is shown in Appendix D of the ms.

Appendix

The deconvolution of a two-mode histogram is shown as constructed. Here use a trial and error procedure. In the ms it is done by Tarantola inversion.



The blue histogram (H) is the measured histogram. The histogram in I is the first guess of a deconvolved histogram with 100 and 50 tracks in each column. F and C are the two columns of H. B and E are normalized filters based on laboratory annealing experiments (Green et al., 1987). A is (numbers in C) X B. D is (numbers in F) X E. The convolved histogram is G = A + D. G is compared with the observed histogram H. The left part is not similar. The suggested deconvolved histogram I is therefore not successful. The ms explains how to calculate the histogram I directly from G. See below a more successful inversion.



The two- modal histogram H is successfully resolved into two thermal events, despite the overlap of tracks observed in A and D. The number of tracks in the two columns of I correspond to equivalent time intervals as shown in the ms.