Peter Klint Jensen

Final answer to review by Green

General answers to Green

We disagree with Green on the physical-mathematical understanding of fission-track age dating. Green find that track length annealing models are necessary for age dating of thermal events. In contrast, we show that the ages can be calculated based on surface track density and track length distribution histograms without any use of annealing models. Our view is supported by the equations given by Bertagnolli et al., Keil et al., Jensen et al., and the present ms by Jensen and Hansen. A simplified model for the development of fission tracks presented below supports our view.

Green believes that the initial spread and further broadening of track lengths during annealing exclude the possibility to "pick apart" the observed track length distribution to assign fission-track ages to each track length bin. We are *not* assigning an age to the bins of observed fission-track length histograms. Ages are assigned to the bins of the *de-blurred* (deconvolved) track length histograms which then are used to age date thermal events.

Green presents an example of a thermal history leading to a track length histogram with two populations. Green finds that the considerable spread of track lengths excludes the possibility of distinguishing between tracks belonging to the two populations. We agree that this is not possible when considering individual tracks of an observed track length histogram. But it can be done on bin level for *de-blurred* track length histograms. The tracks of the observed track length histogram (on bin level) can then be assigned to the two populations with uncertainties indicated by error bars. That is, the large spread of track lengths during annealing can to some extent be removed. This is possible because the spread of tracks known from laboratory annealing experiments is not completely coincident. In an example, we show that tracks from two populations can be distinguished despite the large spread.

A recipe for age dating thermal events and calculating temperature history is given to clarify when there is a need for an annealing model and when not. We consider an idealized model with tracks generated continuously (not spontaneously), no length spread during annealing, and no errors of measurements.

1. Construct a track length histogram of the randomly oriented unetched tracks.
2. Convert the track length histogram into a histogram of time intervals by dividing the number of tracks by the rate of track generation. Each column of this histogram represents the time it takes to generate the number of tracks within the corresponding track length bin.
3. Cumulate the time intervals from the youngest (longest tracks) to the oldest (shortest tracks) to achieve the track ages. A jump of the ages in this histogram dates a marked temperature change. The histogram of the pre-event tracks can now be extracted from the left part (shortest tracks) of the cumulated ages and the post-event histogram from the right part (longest tracks). Post-event tracks are identified using the cumulated age. To this point, there has been no need for a track annealing model.

4. Calculate the temperature history based on the histogram of time intervals. Choose for example a piecewise linear function or a step function. Start with the rightmost column. The first age-temperature point is at age zero and present temperature. The next point back in time is at the age given by the rightmost column of the histogram. Find the temperature at this point using an annealing law by adjusting a trial temperature until the track with initial length is annealed to the length given by the leftmost boundary of the bin. This temperature is the temperature at the right boundary of the next bin to the left. The temperature history is thus calculated backward in time. If the age of the thermal event is available from other sources this age is used to separate the histogram into three parts. The rightmost part consists of columns with ages less than the given age. The next column with older tracks contains mixed pre-and post-event tracks. The next older columns are of pre-event origin. The thermal history can be calculated for each part separately.

This recipe is based on the development by Bertagnolli (1983), Keil et al. (1987), and Jensen et al. (1992).

The recipe for the realistic case is presented next. Here the confined horizontal etched track length histogram is considered together with the surface track density. It is assumed that tracks are produced spontaneously, track lengths are spread during annealing, observed track lengths are biased, and measurements are uncertain.

1. Construct a track length histogram of the observed confined horizontal etched tracks.

2. Remove the spread by deconvolving (de-blurring) the measured histogram. The resulting deconvolved histogram is then the basis for treatment similar to the idealized tracks.

3. Convert the deconvolved track length histogram into a histogram of time intervals. The procedure is presented in the ms.

4. Cumulate the time intervals from right to left to achieve the ages of the track of the deconvolved histogram.

5. Jumps observed in the cumulated ages show the presence of temperature events. The ages of these events are then directly read from the cumulated age curve.

6. The part of the observed (measured) histogram with tracks generated before the thermal event is identified by convolution of the left part of the deconvolved histogram. Convolution spreads the idealized tracks to simulate a measured track length histogram. For sedimentary samples, the post-sedimentary part of the measured histogram is be identified similarly. To this point, there has been no need for a track annealing law.

7. The temperature history is calculated from the deconvolved histogram as described for the idealized tracks.

The procedure for calculating the temperature history based on the deconvolved track length histogram was presented by Jensen et al. (1992). At that time, deconvolution was performed by trial and error. Later mathematical simulated annealing was introduced instead (Jensen and Hansen, 2018). Age dating and identifying inherited tracks using Tarantola inversion are presented in the present ms. Calculations are direct with no use of Monte Carlo simulation.

Peter Klint Jensen

Specific answers to Green

Wording:

I suggest a new title: Age dating thermal events by de-blurring horizontal confined fission-track length histograms.

The wording ”Age distribution of fission tracks” is imprecise and should be changed. It must be made clear when we are talking about the observed track length histograms and when we are talking about the de-blurred histograms. We are not age dating tracks of the observed histograms except for the expected age of the oldest track. For the de-blurred histograms, we can use “the age of tracks belonging to the de-blurred histogram”.

L. 26-27. “independent of any annealing law”:

The annealing properties of the apatite mineral affect the final apparent age as well as the expected age of the oldest randomly oriented unetched track. However, the expected age of this track can be determined by counting the number of all tracks in a volume and divide by the track generation rate (eq. 1 in the ms). In the simplified case of no spread of tracks, the expected age of a given track is calculated by counting the number of shorter tracks and add 1. This age is determined without the use of any annealing law. The unetched tracks are not routinely measured and etched confined horizontal tracks are measured instead. The math in the ms explains how they can be used together with the surface track density instead of the unetched tracks.

“pick apart”

After the formation of tracks, the spread of lengths increases during annealing leading to a considerable mixing of lengths of the combined track length histogram. There is not a one-to-one relationship between time and individual tracks. However, the observed track length histogram is not completely blurred. There is a tendency that the oldest tracks appear in the short track length part of the histogram and the newest tracks in the opposite part. De-blurring by deconvolution can to some extent reduce the spread. Deconvolution is used extensively in seismic processing where it is used to increase the signal-to-noise ratio. This requires the character of the noise to be well known. The “noise” in connection with fission tracks is the spread observed in laboratory annealing experiments. This spread is used to reduce the spread of the observed track length histograms. The resulting histogram is the de-blurred or deconvolved histogram. This idealized histogram consists of tracks ordered in accordance with length. The age of the oldest track of each bin of this idealized histogram is determined by counting tracks with no use of an annealing law.

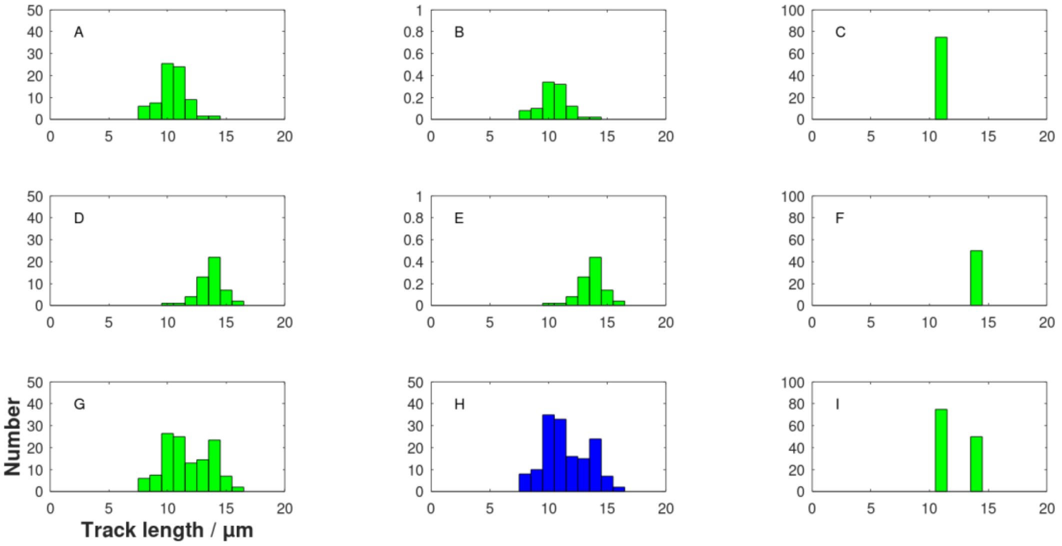
Green presents an example of a thermal history leading to a track length histogram with two overlapping populations. Deconvolution can resolve the overlap and, in this way, determine the timing of the thermal events. However, there are limits to the resolution. If the two populations are too close to each other it is not possible. But, in the example given by Green, it is possible because the overlap is not complete. The deconvolution procedure is illustrated in Appendix 1 below. In Appendix 2 we use our inversion program to identify a thermal event like the example given by Green. The result of a “pick apart” of a two-population 3D track length-angle histogram is shown in Appendix D of the ms.

Conclusion

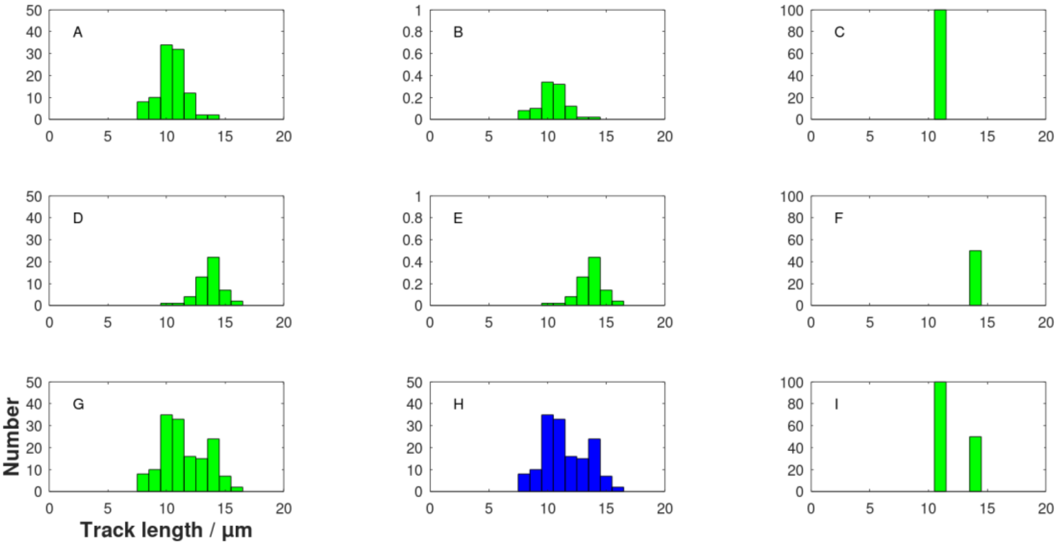
1. The deconvolution method can de-blur a measured track length histogram. Marked thermal events can then be made visible and their ages constrained, however with large uncertainties. There is no need for track-length annealing models in this process.
2. Parts of a measured track length histogram can be identified as being associated with the thermal event, however with large uncertainties.
3. The number of age nodes and their ages can be determined by the deconvolution method.
4. The age nodes are the prerequisite for temperature calculation. The temperature history in between the nodes cannot be derived from the deconvolved fission-track histogram due to the overlapping of the error bars of potential age nodes. Additional information from geological interpretations and other thermal indicators are then needed.
5. We have presented a new 3D representation of fission-track length and angles to the c-axis which increases the information available for temperature history calculation.

Appendix 1

The deconvolution of a histogram with two populations is explained here. The procedure is based on trial and error. In the ms, it is done by least-square Tarantola inversion.



The blue histogram (H) is the measured histogram. The histogram in I is the first guess of a deconvolved histogram with 100 and 50 tracks in each column. F and C are the two columns of H. B and E are normalized filters based on laboratory annealing experiments. A is the histogram in C times the histogram in B. Similarly for D. The convolved histogram is G = A + D. G is compared with the observed histogram H. The left part is not similar. The suggested deconvolved histogram I is therefore not successful. The ms explains how to calculate the histogram I directly from G. See below a more successful inversion.



The two-population observed histogram H is successfully resolved into two thermal events as seen in I. This is done despite the overlap of tracks in A and D. The number of tracks in the two columns of I is then used to calculate the time it takes to generate them.

Appendix 2

The deconvolution method is a mathematical method capable of reducing the length spread of confined horizontal fission tracks. We show that our computer program can de-blur a track length histogram like the one put forward by Green. Assume that a sample starts at a temperature of 63 deg. C, 150 Ma back in time. At 50 Ma it is suddenly uplifted to a temperature of 20 deg. C until the present. The corresponding track length histogram is calculated forwardly by our basin model resulting in the histogram seen in fig. 1 below. A track length annealing model by Stephanson is used. The histogram is like a measured histogram. The resulting two populations of track lengths are hardly seen in the histogram. However, after deconvolution they are revealed, fig. 1b. The deconvolution works because the filters used are not completely covering each other. The filters are derived from laboratory track annealing experiments. In this case, nine filters have been in play. Each column of the deconvolved histogram (fig. 1b) is now converted to the time it takes to generate the tracks belonging to them. The number of tracks and time is almost linearly related. The math in the ms gives a precise relationship. The cumulated age histogram in fig. 1c. is obtained by adding together the time intervals from the most recent to the oldest. The line at 50 Ma shows the time of a predefined rapid uplift used in the forward modeling. Imagine that the timing of this uplift is known from other sources. The post-uplift tracks of the deconvolved histogram are identified based on the error bars to be the four rightmost red columns of the cumulated ages, fig. 1c. Their ages are well below the age of the deposition. Similarly, the green columns contain the post-uplift tracks. The leftmost green column from 5 to 9 micr. m. contains pre-uplift tracks. The column from 9-10 micr. m contains a mix. The post-uplift columns can now be picked from the deconvolved histogram, fig. 1d. This histogram is now convolved (spread out) using the earlier mentioned filters to identify the post-uplift part of the observed histogram with error bars, fig. 1d.

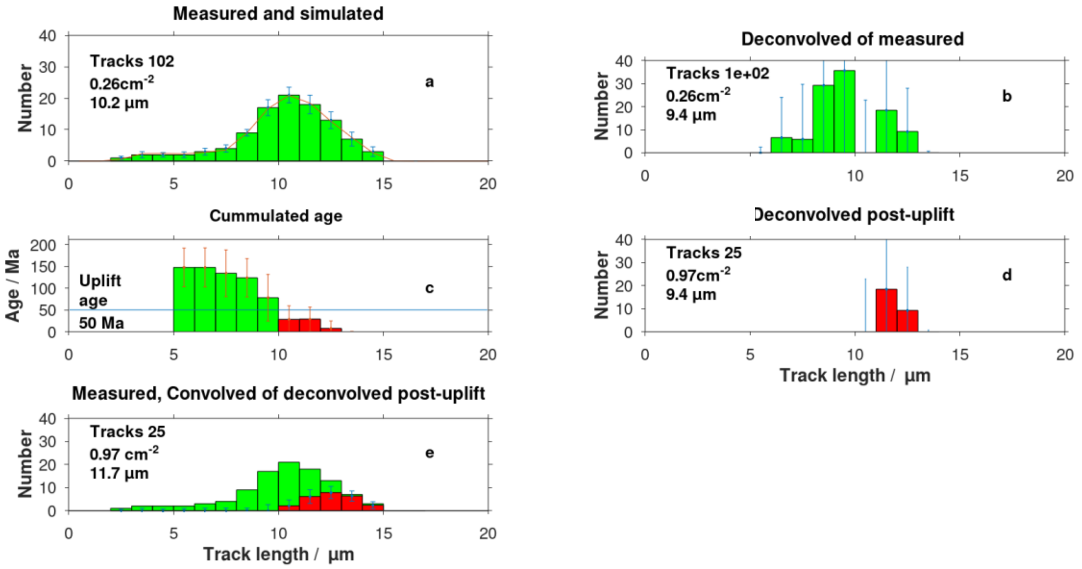
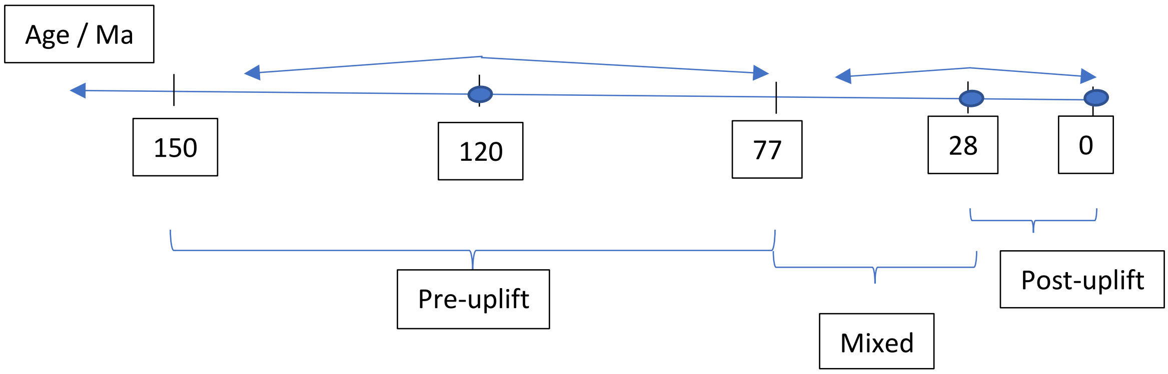


Fig. 1. a. is the forward simulated track length histogram imaging a measured histogram. b. The deconvolved histogram is a de-blurred version of the measured histogram. c. The columns of the histogram in b are converted to time intervals and then cumulated from right to left to obtain the cumulated age histogram in c. d. The identified post-uplift tracks of the deconvolved histogram. e. Convolution (spreading out) of the deconvolved histogram simulates the post-uplift part of the measured track length histogram.

Age dating is the prerequisite for temperature history calculation from fission-track data (standalone). In the example given above three age nodes can be used for temperature calculation (blue circles):



The nodes are chosen so that they are separated from each other by at least one sigma (error bars are read on the cumulated age histogram fig. 1c). The 120 Ma node can only be used provided that there is only one source area.