25. oktober 2021

Authors answers to referee Per Hansen

Line numbers refer to the new version of the manuscript. Some changes in the computer programmes have been made to bring them in accordance with the manuscript. The new versions are updated in the repository.

Referee:

"I think it is perfectly valid to use the Bayesian framework in this paper; but I think the presentation on page 8 will benefit from adding a few more details and a reference to a statistics book."

Author:

Two references on statistics are added to the reference list, l. 561-566.

Referee:

"more details the solution to the problem

$$\min_h \; || \, G\,h - d\text{obs} \, ||_{CD}^2 + || \, h - h_{\text{prior}} \, ||_{CH}^2$$

where I use the notation $|| \, z \, ||_X^2 = z^T X^{-1} z$. This formulation clearly shows that *h* is a regularized solution, with a data-fitting term that involves the covariance matrix *CD* for the noise plus a regularization term, or prior, that involves the covariance matrix *CH* for the solution. As mentioned above, both of these terms come from the above-mentioned Gaussian assumptions about the data and the prior, and the connection to the Bayesian formulation is clearer."

Author:

l. 190-195. New text and Eq. (23, new number) are added to show that the solution is regularized and to strengthen the relationship to the Bayesian formulation.

Referee:

"Equation (23) can be unstable for numerical computations, in which case it is recommended to instead compute the maximum-likelihood estimator as the solution to the linear least squares problem

$$\left\| \begin{pmatrix} L_D^{-1} G \\ L_H^{-1} \end{pmatrix} h - \begin{pmatrix} L_D^{-1} d_{obs} \\ h_{prior} \end{pmatrix} \right\|_2^2$$

where *LD*-1 and *LH*-1 are the Cholesky factors of *CD* and *CH*, respectively. Whether this is an issue here depends on the condition numbers of the matrices *G*, *CD* and *CH*; it may be relevant to reflect on this."

Author:

l. 399-406. New text is added here to reflect on stability.

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Referee:

“I am not sure I understand what the authors mean by “Modelization variance caused by the

variance of $\mathbf{G}$” …. please elaborate on this and preferably give equations.”

Author:

l. 207-214. New text and Eq. (27) are added to explain modelization variance.

Referee:

“Moreover, I cannot find any discussion or expression for the covariance matrix *CH*, ….. I do not understand the sentence. Please explain how C_H is computed, such that the experiments (in principle) can be
reproduced.”

Author:

l. 216-225. These new lines with equations explain how C_H is computed.