



A revised alpha-ejection correction calculation for (U-Th)/He thermochronology dates of broken apatite crystals

John He¹, Peter W. Reiners²

¹Department of Geoscience, University of Arizona, Tucson, AZ, 85721, USA

²Faculty of the Environment, University of Northern British Columbia, Prince George, BC V2N 4Z9, Canada

Correspondence to: John He (johnhe@email.arizona.edu)

Abstract. Accurate corrections for the effects of alpha ejection (the loss of daughter He near grain or crystal surfaces due to long alpha-stopping distances) is central to (U-Th)/He thermochronometry. In the case of apatite (U-Th)/He dating, alpha-ejection correction is complicated by the fact that crystals are often broken perpendicular to the c-axis. In such cases the correction should account for the fact that only some parts of the crystal are affected by alpha-ejection. A common current practice to account for such broken crystals is to modify measured lengths of broken crystals missing one termination by a factor of 1.5, and those missing both terminations by a factor of 2. This alpha-ejection “correction correction” systematically overestimates the actual fraction of helium lost to alpha ejection, and thus overcorrects the measured date relative to that determined for an otherwise equivalent unbroken crystal. The alpha-ejection-affected surface-area-to-volume ratio of a fragmented crystal is equivalent to the surface-area-to-volume ratio of an unbroken crystal twice as long (for fragments with one termination), and equivalent to that of an unbroken crystal infinitely long (for fragments with no termination). We suggest it is appropriate to revise the fragmentation correction to multiply the length of crystals missing one c-axis termination by 2, and those missing both c-axis termination by some large number $> \sim 20$, respectively. We examine the effect of this revised correction and demonstrate the accuracy of the new method using synthetic datasets. Taking into account alpha-ejection, rounding of the He concentration profile due to diffusive loss, and accumulation of radiation damage over a range of thermal histories, we show that the revised fragmentation alpha-ejection correction proposed here accurately approximates the corrected date of an unbroken crystal (“true” date) to within $< 0.7\%$ on average ($\pm 4.2\%$, 1σ), whereas the former method overcorrects dates to be $\sim 3\%$ older than the “true” date, on average. For individual grains, the former method can result in older dates by a few percent in most cases, and by as much as 12% for grains with aspect ratio of up to 1:1. The revised alpha-ejection correction proposed here is both more accurate and more precise than the previous correction, and does not introduce any significant systematic bias to the apparent dates from a sample.

1 Introduction

Since the development of modern apatite (U-Th)/He thermochronometry, the technique has become a versatile and powerful tool for a range of geological problems (Zeitler et al. 1987; Farley *et al.*, 1996; Flowers et al., in press a, b). To fully leverage the power of the technique, however, it is necessary both to account for the wide range of possible complications that commonly cause data dispersion greater than analytical errors. Of particular significance is correcting for the loss of daughter nuclides due to the problem of alpha ejection. Apatite He dating uses the accumulation of daughter nuclide ⁴He (i.e., alpha particles) from the spontaneous alpha decay of ²³⁸U, ²³⁵U, and ²³²Th (as well as a minor contribution from ¹⁴⁷Sm) to constrain possible thermal histories of samples, which is sometimes simplified as providing a date of cooling through some closure temperature (~ 30 - 90 °C) at which helium diffusion out of a crystal is sufficiently slow for the system to be considered closed. However, this method is complicated by the fact that the sizes of most typical apatite crystals are only



several times greater than the stopping distance of alpha particles (~20 μm), meaning that the fraction of ⁴He ejected from a crystal must be accounted for in most applications (Farley *et al.*, 1996).

Careful measurement of crystal geometries allows accurate approximation of the cumulative alpha-ejection loss of helium from a crystal (Ziegler; 1977; Farley *et al.*, 1996; Farley, 2002; Hourigan *et al.* 2005; Ketcham *et al.* 2011; Reiners *et al.*, 2018). Because the likelihood of an alpha particle being ejected from a crystal is directly related to a parent nuclide's proximity to the crystal surface, the fraction of helium retained in the crystal (F_T) is a function of a crystal's surface area to volume ratio (β) (Farley *et al.*, 1996). In the simplest case of a spherical grain with homogenous parent nuclide distribution, F_T is a cubic polynomial function of β (Farley *et al.*, 1996). F_T can be estimated for other geometries using a polynomial function calibrated by Monte Carlo alpha-ejection models (Farley, 2002; Ketcham *et al.*, 2011). In practice, a parent-nuclide-specific F_T^i is determined and a corrected date can be calculated by incorporating it into the full decay equation:

$${}^4\text{He} = 8({}^{238}\text{U})(F_T^{238})(e^{\lambda_{238}t} - 1) + 7({}^{235}\text{U})(F_T^{235})(e^{\lambda_{235}t} - 1) + 6({}^{232}\text{Th})(F_T^{232})(e^{\lambda_{232}t} - 1) + ({}^{147}\text{Sm})(F_T^{147})(e^{\lambda_{147}t} - 1)$$

50 [Eq. 1]

where t is the unknown variable that must be solved numerically or iteratively; ⁴He, ²³⁸U, ²³⁵U, ²³²Th, and ¹⁴⁷Sm contents are measured; λ_i is the decay constant for the given isotope; F_T^i , the alpha-ejection correction factor for the given isotope calculated from crystal geometry (Ketcham *et al.*, 2011). A more approximate corrected date can also be calculated by simply dividing the measured (raw) date by F_T (Farley and Stockli, 2002), though this is less accurate for older dates.

55 These calculations generally assume the ideal case of a euhedral, prismatic crystal with homogenous parent nuclide distribution, entire original crystal faces, and insignificant parent nuclide concentrations outside and within one alpha-stopping distance of the exterior of the crystal during the interval in which temperature was low enough to accumulate He. When these assumptions are violated, further adjustments to the standard F_T correction are required.

If information about the magnitude and pattern of parent-nuclide zonation is available, an adjusted F_T may be applied to account for inhomogeneous parent-nuclide distribution (Hourigan *et al.*, 2005, Farley *et al.*, 2011, Ault and Flowers, 2012, Gautheron *et al.*, 2012). Absent such information, as is the case in most routine analyses, use of the standard unzoned correction assuming homogenous distribution of the parent nuclide would introduce errors that skew a crystal's apparent date to be younger, if its rim is enriched in parent nuclides, and older if it is depleted in parent nuclides (Farley *et al.*, 1996; Hourigan *et al.*, 2005). For apatite, these errors are usually minor (<1.5% for 80% of apatite crystals, and <9.5% for 95%), because apatite crystals in most cases do not typically exhibit extreme zonation of parent nuclides (Ault and Flowers, 2011). Furthermore, the errors are usually symmetrically distributed, with apatite populations not exhibiting bias towards either rim-enriched or rim-depleted grains (Ault and Flowers, 2011). Accounting for effects of He implantation from sources external to the grains is not typically possible for grains separated from their petrographic context, although in some cases particular date-eU or date-size correlations may be used to interpret such effects (Spiegel *et al.*, 2009; Murray *et al.*, 2014).

70 The focus of this paper is the adjustment to the F_T correction that should be made in the case of crystals that are broken perpendicular to the c-axis, as common for apatite. If errors due to fragmentation are large, they can significantly impede our ability to extract geologically meaningful information from dates calculated from parent-daughter nuclide ratios. Broken and suboptimal crystals are frequently analyzed, particularly when the quality of mineral separates is poor and/or the apatite yield from a sample is low. In addition, imperfect basal (0001) cleavage in apatite (Dana, 1963; Palache *et al.*, 1963) leads to the fact that many dated crystals are broken perpendicular to their c-axis and lack original terminations, even for high-quality



samples. Assuming that fragmentation occurred recently relative the date measured (e.g. during mineral separation, or in the case of detrital samples, during recent transport), a common strategy is to apply a fragmentation correction to the F_T calculation, which accounts for the fact that the fracture exposes surface area where alpha ejection did not occur (Farley, 2002). This correction seeks to approximately correct for the originally greater length of the unbroken apatite crystal, by
80 multiplying the length of all broken crystals by 1.5 (if one end is broken) or 2 (if both ends are broken) (Farley, 2002; Farley *et al.*, 1996; Brown *et al.*, 2013; Beucher *et al.*, 2013; Reiners *et al.*, 2018). Though it is not possible to find the original length of broken crystals, Farley (2002) argued that these approximations are sufficient because F_T is relatively insensitive to the length of the crystal.

An alternative approach to this problem is that of Brown *et al.* (2013), who argued that, for interpreting thermal histories, it
85 is best to leave dates uncorrected and instead evaluate the variation in date among crystals with different morphologies and numbers of broken ends. If one assumes that breakage occurred prior to cooling to temperatures of partial He retention, raw (uncorrected) dates of broken crystals can vary by up to 60% for certain t-T histories, and that for sufficiently large datasets of fragmented crystals, considering the patterns of dispersion in uncorrected dates can constrain thermal history (Brown *et al.* 2013). In practice, however, F_T -corrected dates remain widely reported, partly because correcting for alpha ejection and
90 fragmentation is necessary to compare dates to other datasets and dates of geologic significance. A more accurate correction would allow both broken and unbroken crystals within a sample and across samples to be appropriately compared without introducing additional systematic bias.

Although the conventional fragmentation correction has been widely applied since the widespread application of the
95 technique, its accuracy and precision has not been demonstrated. In the first part of this paper, we consider the rationale behind the early approach, then propose a revision and compare the results of both methods. We then test the new method using synthetic data and demonstrate the accuracy of the revised correction. We take into account a range of broken crystal sizes, number of terminations present, and various thermal histories and their associated effects on helium diffusivity, and we quantify the uncertainty that can be attributed to the fragmentation correction alone. Considering the numerous natural
100 sources of uncertainty in apatite He dating, achieving greater confidence in the accuracy of the fragmentation F_T correction and minimizing its uncertainty ultimately aids in the interpretation of other possible sources of uncertainty and errors (He *et al.*, 2021).

2. Revision of F_T correction for broken crystals

For an idealized spherical grain, the alpha-ejection correction is a function of the radius of the sphere (R) and the alpha
105 stopping distance for the given parent nuclide (S):

$$F_T = 1 - \frac{3S}{4R} + \frac{S^3}{16R^3} \quad [\text{Eq. 2}]$$

(Farley *et al.*, 1996). Where $R \gg S$, the function approaches a linear relationship:

$$F_T = 1 - \frac{3S}{4R} \quad \text{or,} \quad F_T = 1 - \frac{S\beta}{4} \quad [\text{Eq. 3}]$$

110 (Farley *et al.*, 1996), where β is the ratio of surface area of a crystal to its volume. In other words, the fraction of helium lost due to alpha ejection near the crystal surface is approximately a function of the ratio of surface area of a crystal to its



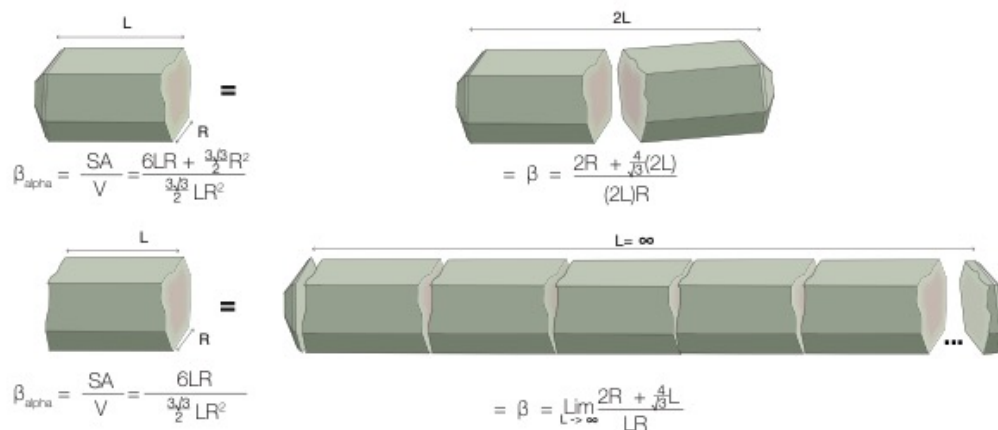
115 volume. Considering more realistic crystal geometries, polynomial equations that define the F_T^i value as a function of β have been empirically determined using Monte Carlo simulations for each parent nuclide i and their respective alpha stopping distance (Farley et al., 1996; Hourigan et al., 2005). For hexagonal prisms, simply measuring the length (L) and radius or half-width of the cylindrical prism (R) allows the computation of β :

$$\beta = \frac{(4/\sqrt{3})L + 2R}{LR}$$

[Eq. 4]

120 The general idea behind modified F_T corrections is to modify β , under the assumption that the polynomial functions relating β and F_T are nearly identical for similar geometries (e.g., hexagonal prism with bipyramidal or pinacoidal terminations). This was the approach taken to correct for lost crystal surface in the case of crystals polished parallel to the c-axis (Reiners et al., 2007). In the case of c-axis perpendicular breakage, the Farley et al. (1996) approach sought to establish the length of the original, unbroken crystal. Because it was observed that the corrected-length-to-radius ratios of most apatite crystals (5:1) were sufficiently high such that the F_T corrections become largely independent of length, it became standard practice at most
125 laboratories to simply modify β by multiplying the lengths of broken crystals by arbitrary factors of 1.5 or 2, thereby modifying F_T (Farley *et al.* 1996; Farley, 2002). Alternatively, in the slightly different context of inverse modelling a large set of uncorrected ages, Beucher et al. (2013) suggested that a rule of thumb for predicting the unknown initial length should be to add the maximum fragment length of a set of fragments and two times the maximum radius. To the first order, guessing the unknown initial length using consistent but arbitrary factors such as these suffices to roughly account for the
130 loss of alpha-ejection-affected surface area at the tips: this is because as L increases, the increase in surface area of a crystal is less than that of the volume, in effect reducing β .

In detail, however, the fraction of helium remaining in a fragmented crystal does not depend on the unknown (and precisely unknowable) initial length. Rather, it should be directly related to the surface area of a broken crystal that was originally affected by alpha-ejection. Assuming that we can identify when crystals have lost one or both terminations, that the breakage
135 generally occurs more than one average-alpha-stopping-distance from the tip, and that diffusion has not significantly modified the daughter concentration profiles, the alpha-ejection-affected-surface-area-to-volume ratio (β_a) could be simply calculated by measuring and then subtracting the surface area of the broken face(s) from the total surface area. For a hexagonal prism, simple geometric calculations demonstrate that the β_a of singly and doubly broken crystals are equivalent to the β of an unbroken crystal of the same width that is twice as long, or infinitely long, respectively; this is shown
140 graphically in Fig. 1. Consider that the helium profile in an unbroken crystal is symmetrical such that when broken in half, each half will have the same fraction of helium remaining (F_T); thus, conversely, any broken crystal with one termination (breakage occurring more than one alpha-stopping distance from the tip) has the same F_T as a hypothetical unbroken crystal double its length (but not the same as the original unbroken crystal). For crystals with no terminations remaining, any c-axis perpendicular segment or cross section of the crystal will have the same F_T , no matter its length or its position along the
145 fragment; therefore fragments with no terminations have the same F_T as an infinitely long unbroken crystal, where the terminations, which have a different F_T , have a vanishingly small effect on the overall F_T of the crystal.



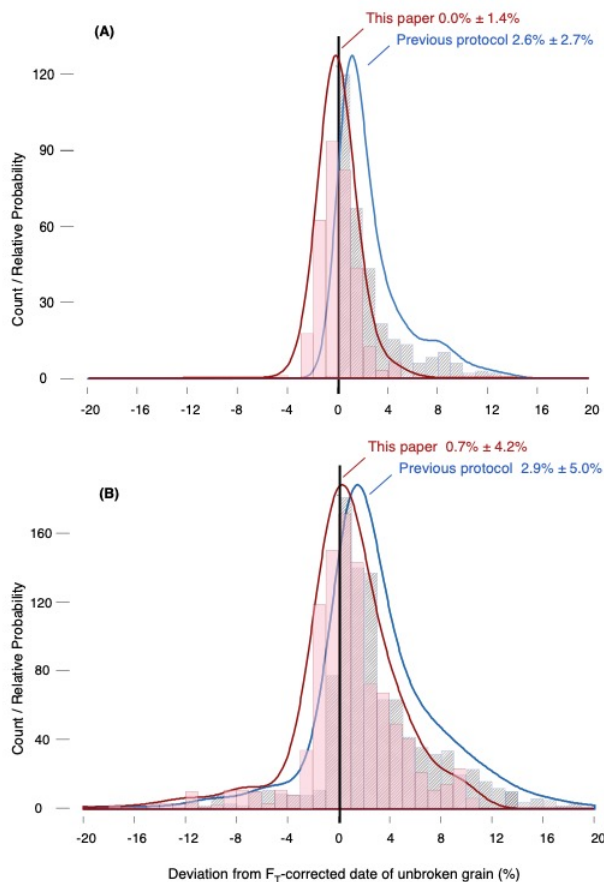
150 **Fig. 1. The alpha-ejection-affected surface-area-to-volume ratio, or β_{α} , of a broken crystal that has lost a basal fragment longer than one alpha-stopping distance is equivalent to the surface-area-to-volume ratio of an unbroken crystal twice as long (for fragments with one termination), and equivalent to that of an unbroken crystal infinitely long (for fragments with no termination). The simple geometric calculations shown assume a flat hexagonal termination, but apply by the same logic to any geometry, regardless of the shape of its body (e.g. cylindrical, tetragonal, or hexagonal) or the shape of its terminations (e.g. pyramidal, flat, or rounded). This assumes that the fragment is broken more than one average-alpha-stopping-distance from the tip, and ignores any change to the daughter concentration profile due to diffusion.**
 155

It follows, then, that a more accurate fragmentation correction that explicitly considers the lost surface area of a broken crystal should be to multiply the length of a broken crystal by 2 or some large number (to simulate the limit as L approaches infinity), respectively, rather than 1.5 or 2, i.e.:

160
$$\beta_{\alpha} = \frac{(8/\sqrt{3})L + 2R}{2LR}$$
 for crystals broken on one end; and

$$\beta_{\alpha} = \lim_{L \rightarrow \infty} \frac{4/\sqrt{3}L + 2R}{LR}$$
 for crystals broken on both ends

165 where L is the measurement of the crystal dimension perpendicular to the fracture (since crystals commonly break perpendicular to the c-axis in apatite crystals, L should usually be measured parallel to the c-axis, even if it is shorter than the width). This simple correction has the benefit of applying to other geometries, regardless of the shape of its body (e.g. cylindrical, tetragonal, or hexagonal) or the shape of its terminations (e.g. pyramidal, flat, or rounded): the β_{α} of any singly broken crystal is equivalent to the β of a whole crystal twice its length, and the β_{α} of any doubly broken crystal is equivalent to the β of a whole crystal infinitely long. The correction can thus be applied to all crystals broken perpendicular to the c-axis, regardless of original length, requiring knowledge of only the width and length of the broken crystal, and the number of
 170 terminations present. We emphasize that though this fragmentation correction is similar in form to that of Farley (2002) in that it involves length-modifying factors, it differs in that it seeks to approximate the length of a whole crystal with the same fraction of helium remaining as the fraction remaining in the broken fragment, rather than seeking to approximate the unknown length of the original unbroken crystal.



175 Fig. 2. Histograms and kernel density estimate (KDE) functions comparing the new protocol suggested in this paper (red) and the
previous protocol (blue), as applied to a synthetic dataset of raw, uncorrected dates ($n=1000$, cf. Brown et al., 2013). The error is
180 defined to be the % deviation of the F_T -corrected dates of a broken crystal from the F_T -corrected date of the whole crystal, which
is known because the synthetic fragments are generated from a whole crystal with a known “age”. The new protocol is more
accurate and more precise at correcting for the effect of fragmentation, both in the ideal case (A) assuming that fragmentation has
occurred more than one average-alpha-stopping-distance from the tip, and that there is no significant diffusive modification of the
helium profile, as well as in the more realistic scenario (B) when these assumptions are relaxed. Annotations show the mean \pm
standard deviation (1σ); KDE functions are normalized to the same peak height.

3. Accuracy of the revised F_T correction

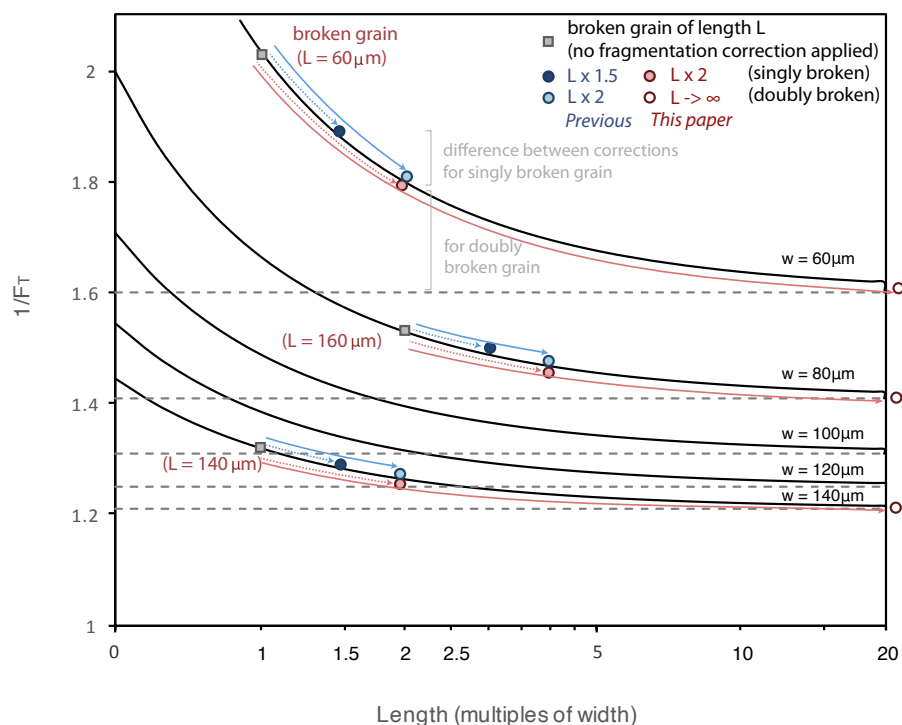
185 The new protocol accurately corrects for the effect of fragmentation, deviating by $0.0\% \pm 1.4\%$ (1σ) from the corrected date
of the unbroken crystal, under the ideal assumptions stated above: that the fragmentation has occurred more than one



190 average-alpha-stopping-distance from the tip, and that there is no significant diffusion-induced modification of the helium
concentration profile (Fig. 2a). By comparison, under the same assumptions, the old protocol leads to corrected dates that are
almost all too old, on average by $2.6\% \pm 2.7\%$. Characterization of the uncertainty for both protocols is based on the
application of the protocols to a synthetic dataset of raw (uncorrected) dates of broken prismatic crystals where the corrected
195 dates of the original unbroken crystals are known. Note that as a simplification, a length-modifying factor of 20 is used to
approximate the limit to infinity (see Section 4.1). For comparable results, we used the same datasets of raw uncorrected
dates from Brown *et al.* (2013), which are generated from the volume-integrated ^4He concentration in a random set of crystal
fragments broken at varying positions along the original crystal (Beucher *et al.* 2013). These original unbroken crystals from
200 which the fragments were generated have a constant geometry (hexagonal prism that is $400\ \mu\text{m}$ long and $150\ \mu\text{m}$ wide). To
test the accuracy of the proposed protocol under the stated ideal assumptions, we excluded any randomly generated crystals
that are $<20\ \mu\text{m}$ from the tip, and included only fragment sets that experienced the two thermal histories associated with the
least amount of diffusive modification of the helium profile. We assume uniform spatial distribution of the parent nuclide,
and apply both protocols to all fragments as we would in routine laboratory analyses: i.e. we assume no knowledge of the
original length and thermal history of the crystals to compute the corrected age. Only the length and width of the broken
crystals, and the number of terminations present, are used for the calculation.

205 Though the new F_T correction for broken crystals is more accurate as a whole than the old correction, the two ideal
assumptions of the simple geometric argument above introduce additional uncertainty. In a more realistic scenario, when the
two assumptions are relaxed, the proposed fragmentation correction results in a broader range of uncertainty ($+0.7\% \pm$
 4.2%), but it is nevertheless more accurate and more precise than the old protocol ($+2.9\% \pm 5.0\%$) (Fig. 2b). Using the new
protocol, only 3% of corrected dates deviate from the corrected date of the unbroken crystal by greater than 10%; this
represents a 66% reduction relative to the prior protocol. These results are based on the full fragment dataset from Brown *et*
al., 2013, which includes a representative range of thermal histories that are more complicated than simple rapid cooling (i.e.
slow, monotonic cooling; prolonged isothermal residence in the partial retention zone followed by rapid cooling; a mix of
210 slow cooling and isothermal holding in the partial retention zone; and gradual reheating (e.g. burial) followed by rapid
cooling; cf. Wolf *et al.* 1998). 23% of the randomly generated fragments in that dataset were broken within $\sim 20\ \mu\text{m}$ from the
tip, accounting for the possibility that in real laboratory analyses, it is not always discernible whether an apatite crystal was
broken more than an alpha-stopping distance from the tip. Thus, our application of the two F_T corrections to the full dataset
in Fig. 2b approximates the actual circumstances under which broken crystals are analyzed.

215



220 Fig. 3. The inverse of F_T value plotted as a function of crystal length L , as used in the calculation of β or β_a . For a broken crystal, this is measured parallel to the c -axis (as a multiple of width), and modified by some factor (see legend). The inverse of F_T can be approximately considered a multiplier for the raw date, where corrected date \approx raw date $\times (1/F_T)$ (Farley et al., 1996). Inverse F_T corrections for three example fragments are shown (width = 60, 80, and 140 μm , and length = 60, 160, and 140, respectively), with the corresponding fragment-corrected F_T values for singly- or doubly-broken crystals. The gray squares indicate the actual length of the fragment, and show their inverse F_T values if no fragmentation correction is applied. Dashed lines indicate the asymptote for $\lim_{L \rightarrow \infty}$. For convenience, the asymptotic value is assumed to be approximately the same as multiplying by some large number (e.g., 20), plotted at the edge of the graph (open red circle). Note that strictly speaking, calculation of a corrected date requires using the full decay equation with an individual F_T^i for each nuclide, rather than the simplification of corrected date \approx raw date $\times (1/F_T)$.

230

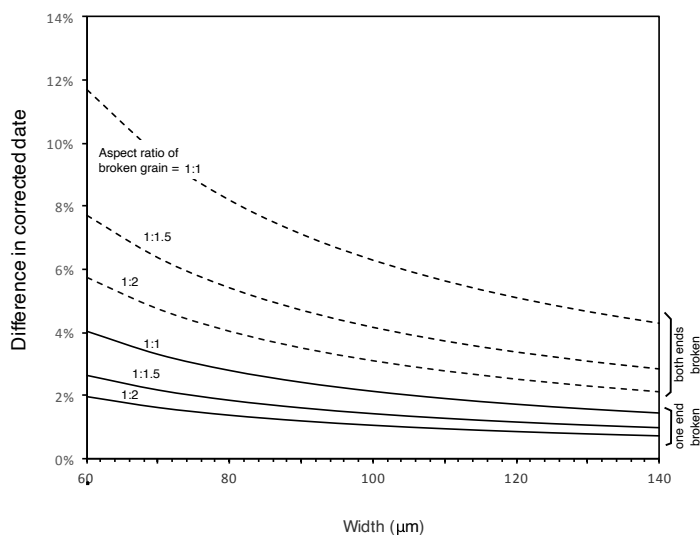


4. Discussion

4.1. Difference between corrections for different crystal dimensions

235 Though multiplying a fragment length by different factors may seem to be a minor revision, the resulting difference in
corrected dates is not negligible. This is partly because the presumption that F_T corrections does not strongly depend on
length, upon which the fragmentation correction was initially based, does not hold true for the smaller crystals commonly
analyzed today (e.g., c-axis perpendicular width $< 150\mu\text{m}$). Fig. 3 shows the effect of crystal length (or modified crystal
length, as used in the calculation of β or β_a) on the inverse value of F_T , an approximation for the correction's effect on the
final reported date (Fig. 3). Since both protocols effectively multiply the length of a broken crystal to compute an adjusted F_T
value, the inverse F_T values of the new and old protocols for any given crystal width all lie on the same curve for F_T as a
240 function of length (normalized to width). Particularly when the length of a broken crystal is close to its width, and when the
width is small, the F_T correction is not independent of the modified length. For example, for a singly-broken crystal that is 60
 μm in width and equally long (the minimum dimensions of crystals routinely analyzed in our lab), the difference between the
new and old protocols, would be 4%; for a doubly-broken crystal of the same dimensions, the difference would be 12% (Fig.
4). The overcorrection of the previous protocol could be even larger for drum-shaped fragments (i.e. crystals broken on both
245 ends, and shorter in c-axis-parallel length than width). For a broken crystal that is 140 μm in both width and length, the
difference would be 2% and 5% (for singly and doubly-broken crystals, respectively). The magnitude of these differences is
not negligible, at least relative to other sources of error in F_T corrections. By comparison, for example, the updated alpha-
ejection models of Ketcham *et al.* (2011) based on revised alpha-stopping distances affects dates by approximately 1-5%,
and 2D measurement of crystal geometry introduces errors of $\sim 2\%$ (Cooperdock *et al.*, 2019).

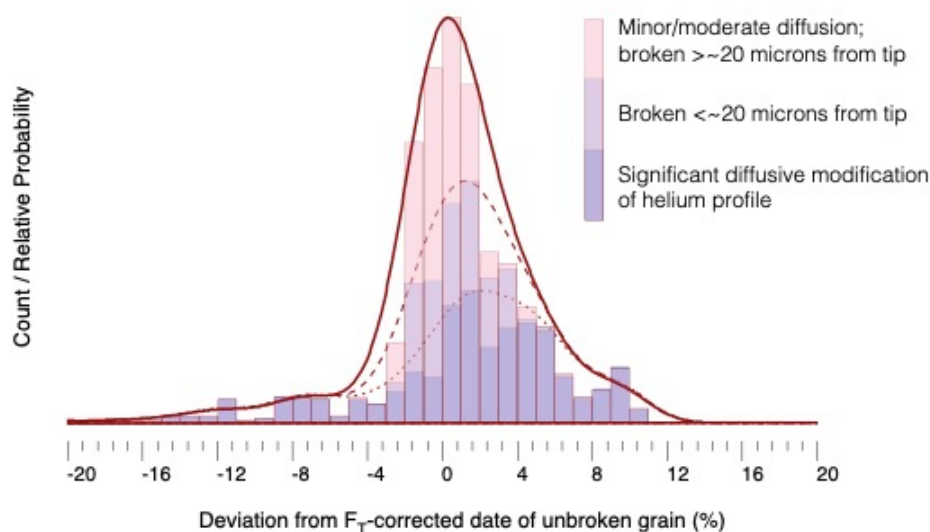
250



255 **Fig. 4.** Difference between the corrected date calculated from the standard protocol and revised protocol (Dashed—broken on both ends; Solid—broken on one end), shown for a range width-to-length ratios commonly seen in broken crystals.

4.2. Uncertainty in fragmentation correction compared to other sources of date dispersion

260 The larger uncertainties in the fragmentation correction with the two ideal assumptions relaxed (Fig. 2b) are largely due to the diffusive modification of helium profiles. In our test of this protocol, all cases of corrected fragment dates that deviate by more than >5% from the corrected date of the unbroken crystal can be attributable to thermal histories involving prolonged residence in the partial retention zone (Fig. 5). Without a priori knowledge of a sample's thermal history, this is a problem for the new fragmentation correction just as it is for the old protocol, because the calculation of F_T correction only assumes loss of helium due to alpha ejection. The additional uncertainty associated with the fragmentation correction fundamentally relates to the fact that using β_α to correct F_T implies taking the lost surface area ("skin") affected by alpha ejection as a proxy for the lost volume (the outer "shell") of the crystal affected by alpha ejection.
265



270 **Fig. 5.** Uncertainty associated with the proposed fragmentation correction due to inclusion of fragments broken too close to the tip (dashed) and due to thermal histories that involve significant diffusive modification of the helium profile (dotted). The histogram and curves are stacked and cumulative, such that the dashed probability curve includes both close-to-tip fragments as well as fragments with significant diffusive loss.

We emphasize that because the F_T -corrected dates of the fragments are compared to the F_T -corrected date of a whole crystal, Fig. 2 and Fig. 5 assesses only the effect of brokenness correction alone. Zonations, eU variation, and diffusive helium loss remain important sources of additional error and dispersion (e.g. Meesters and Dunai, 2002; Herman et al., 2007, Gautheron et al 2012, Brown et al., 2013; Beucher et al., 2013). Notwithstanding the effects of date variation due to all these effects, the fragmentation correction proposed in this paper more consistently and accurately reproduces the F_T -corrected date of unbroken crystal. An illustrative case is that of a hypothetical date-elevation transect from a crustal block that cooled slowly through the partial retention zone until some point in time, then subsequently experienced very rapid cooling (from Brown et al., 2013). A key observation of Brown et al. (2013) was that the large dispersion of raw uncorrected fragment dates is due to the fact that these dates can be both younger and older than the whole crystal, and that fragments of same length can yield different dates, while conversely, fragments of different lengths can yield the same date. The dispersion is compounded because slow cooling leads to significant diffusive modification of the helium profile in a crystal. Despite this large dispersion of uncorrected fragment dates (up to 60%), and despite variations in eU and grain sizes, applying the new fragmentation correction introduces limited uncertainty relative to the dispersion caused by other effects (Fig. 6). This facilitates interpretation of widely-dispersed data by reducing the number of variables that must be considered, and demonstrates the utility of applying a fragmentation correction when analysis of the pattern of dispersion in >20-30 crystals is not practical. Finally, while both the new and old F_T correction for broken crystals reliably approximates the corrected



date of an unbroken crystal for a range of eU and crystal sizes, the new correction reduces the systematic bias that is introduced by the old protocol when many broken crystals are analyzed in a sample by ~3-4% (Fig. 6).

290

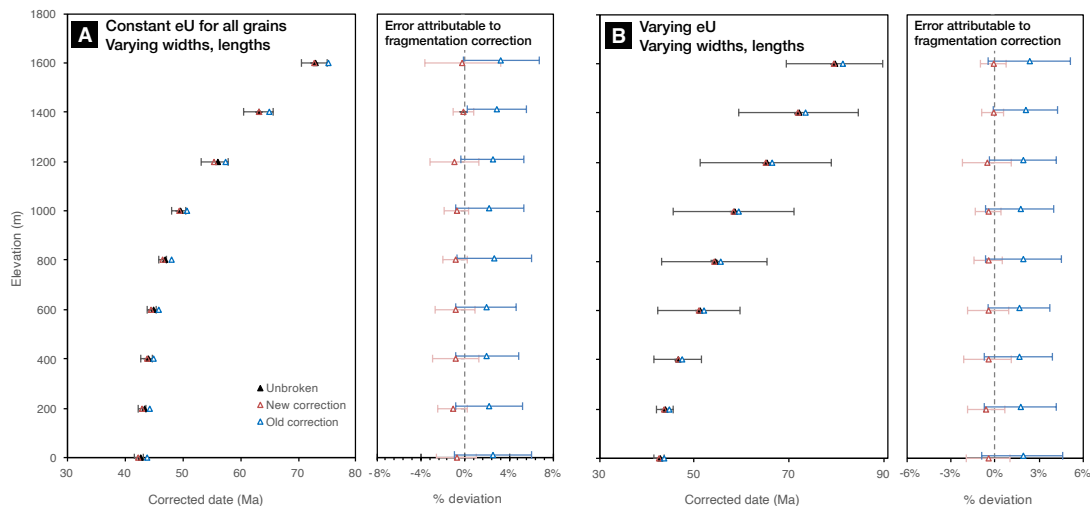


Fig. 6. a) Date-elevation transect of a crustal block that cooled slowly through the partially retention zone, and was subsequently rapidly exhumed (cf. Brown et al. 2013, Fig. 9), showing fragment dates corrected using the new protocol (red), compared to previous protocol (blue), and the expected whole-crystal date (black). Note that the red and black triangles nearly overlap in all cases. Corrected dates are from fragments of varying lengths, both singly and doubly broken, generated from crystals with variable widths and lengths, but constant eU. Dispersion of corrected fragment dates are shown for the new protocol only. Right panel shows the average deviation of the corrected fragment dates from their corresponding corrected whole-crystal dates, using the new (red) and old protocol (blue). (b) Same as (a), but fragments were generated from crystals of varying eU, in addition to varying width and length. Notice that the introduction of eU as a variable significantly increased the dispersion of corrected dates, but that the error attributable to the fragmentation correction alone was not materially affected. All error bars are 1σ .

295

300

Finally, the uncertainty due to the application of the fragmentation correction can be propagated with other sources of error (analytical, zonation, etc.), for appropriate comparison of fragment dates and other corrected dates where the correction was not applied. For larger datasets, especially in the case of larger-n analyses, this contributes to our understanding of the expected distribution of corrected dates from any given sample (Fig. 7) (He et al., 2021). Previous work has shown, for example, that

305

- (1) size- and eU-dependent diffusivity can cause apparent dates to systematically vary (Reiners and Farley, 2001; Flowers et al. 2009; Whipp et al., 2022).
- (2) The presence of extraneous daughter nuclides whose parent nuclides are not accounted for can cause outlier dates many multiples older than the true date (^4He -rich fluid inclusions; U- or Th-rich inclusions or microinclusions where the inclusions are not fully dissolved; grain boundary phases or adjacent grains that

310



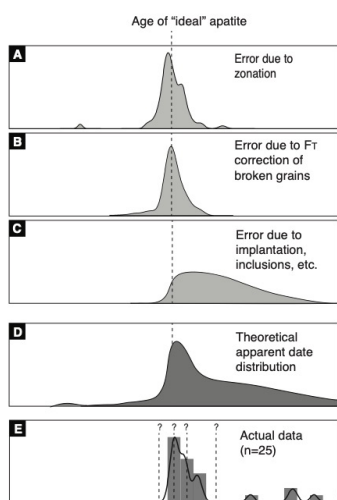
contribute helium to the grain but are not in the analyzed aliquot) (Spiegel et al. 2009; Fitzgerald et al. 2006; Murray et al. 2014).

315

(3) Non-uniform distribution of the parent nuclides can cause dates to be both older or younger by a few percent in most cases. Even if zonations are not accounted for, the probability distribution of errors due to zonations can be approximated by either examining a representative selection of apatite in a sample, or using a reference compilation (e.g. Ault and Flowers, 2011).

(4) Technician-to-technician differences in 2D grain measurement cause date variations that differ from actual 3D geometry by ~2%. (Cooperdock et al., 2019)

320



325

Fig. 7. Schematic illustration of how uncertainties due to different complications in apatite helium dating, e.g. uncertainty due to zonation (Ault and Flowers, 2011)(a), uncertainty due correction for broken crystals, this study (b), and error due to implantation and other effects (c) could together be combined to form a theoretical apparent date distribution (d) that would inform our interpretation of real data and the choice of the appropriate summary statistic (whether the minimum, mean, median, peak date) in representing a sample's date. The last panel (e) is a representative sample (n=25) that shows the expected non-normal and right-skewed distribution from an actual large-n sample (He et al., 2019; Thomson et al., 2019). The peak date would likely approximate the date of an "ideal" apatite, but a full accounting of the probability of various errors would provide a more robust rationale for that interpretation.

330



A future step towards a more rigorous evaluation of the uncertainty of individual-grain analyses or samples as a whole could involve the propagation of each of these uncertainties to account for sample and grain specific information such as the probability of implantation (approximated from the spatial distribution of heavy minerals in a sample, via XRCT), the probability of extreme parent nuclide zonations in a sample (based on fission track mounts) or from a reference compilation, and whether fragments were analyzed.

5. Conclusion

Despite the dispersion of raw U-Th/He dates due to fragmentation, it is possible to accurately correct for the effect of fragmentation based on basic measurements routinely recorded during the grain selection process. In compensating for the effects of alpha ejection in broken crystals, the F_T correction should be calculated by explicitly taking into account the surface area of the broken face, rather than by assuming the unknown length of the original unbroken crystal. In individual cases, especially crystals with smaller width or whose length is less than or around the same as the width, the difference in apparent dates calculated with the two methods can be 12% or greater.

We further applied both the previous and newly proposed protocol for correction of broken crystals to a synthetic dataset. Even taking into account the effects of diffusive loss of helium and breakage close to tips of crystals, the proposed protocol more accurately and more precisely approximates the F_T -corrected date of an unbroken crystal for a range of complex and simple thermal histories. For a crystal of 150um width, the old calculation leads to apparent dates that are on average 3% older than the corrected dates of unbroken crystals, and in certain cases, up to 20%.

The proposed adjustment allows more accurate comparison of data between samples of varying quality, which is common when a mix of different rock types is sampled. The greatest effect will be for samples where the majority of crystals are broken. Though this adjustment is minor in many cases, when applied to entire datasets, it significantly reduces one common source of error in calculations of individual apparent dates, and removes an easily correctable source of systematic bias towards older dates.

Code availability

Kernel density plots were produced using DensityPlotter (<https://www.ucl.ac.uk/~ucfbpve/densityplotter/>)

355 Data availability

Individual corrected fragment dates can be found in the supplementary file, and are calculated using methods discussed in text and applied to the cited synthetic datasets.

Author Contribution (CReDIT taxonomy)

JH – Conceptualization, formal analysis, methodology, visualization, writing-original draft; PWR- Supervision, resources, methodology, writing-review and editing



Competing Interests

The authors declare they have no competing interests.

References

- Ault, A. K. and Flowers, R. M.: Is apatite U–Th zonation information necessary for accurate interpretation of apatite (U–Th)/He thermochronometry data?, *Geochim. Cosmochim. Acta*, 79, 60–78, <https://doi.org/10.1016/j.gca.2011.11.037>, 2012.
- 365 Beucher, R., Brown, R. W., Roper, S., Stuart, F., and Persano, C.: Natural age dispersion arising from the analysis of broken crystals: Part II. Practical application to apatite (U–Th)/He thermochronometry, *Geochim. Cosmochim. Acta*, 120, 395–416, <https://doi.org/10.1016/j.gca.2013.05.042>, 2013.
- Brown, R. W., Beucher, R., Roper, S., Persano, C., Stuart, F., and Fitzgerald, P.: Natural age dispersion arising from the analysis of broken crystals. Part I: Theoretical basis and implications for the apatite (U–Th)/He thermochronometer, *Geochim. Cosmochim. Acta*, 122, 478–497, <https://doi.org/10.1016/j.gca.2013.05.041>, 2013.
- 370 Cooperdock, E. H. G., Ketcham, R. A., and Stockli, D. F.: Resolving the effects of 2D versus 3D grain measurements on (U–Th)/He age data and reproducibility, *Geochronol. Discuss.*, 1–32, <https://doi.org/10.5194/gchron-2019-3>, 2019.
- Dana, E. s.: *A Textbook on Mineralogy*, John Wiley, New York, 1963.
- 375 Farley, K. A.: (U–Th)/He Dating: Techniques, Calibrations, and Applications, *Rev. Mineral. Geochemistry*, 47, 819–844, <https://doi.org/10.2138/rmg.2002.47.18>, 2002.
- Farley, K. A. and Stockli, D. F.: (U–Th)/He Dating of Phosphates, *Rev. Mineral. Geochemistry*, 15, 559–577, <https://doi.org/10.2138/rmg.2002.48.15>, 2002.
- Farley, K. A., Wolf, R. A., and Silver, L. T.: The effects of long alpha-stopping distances on (U–Th)/He ages, *Geochim. Cosmochim. Acta*, 60, 4223–4229, [https://doi.org/10.1016/S0016-7037\(96\)00193-7](https://doi.org/10.1016/S0016-7037(96)00193-7), 1996.
- 380 Farley, K. A., Shuster, D. L., and Ketcham, R. A.: U and Th zonation in apatite observed by laser ablation ICPMS, and implications for the (U–Th)/He system, *Geochim. Cosmochim. Acta*, 75, 4515–4530, <https://doi.org/10.1016/j.gca.2011.05.020>, 2011.
- Fitzgerald, P. G., Baldwin, S. L., Webb, L. E., and O’Sullivan, P. B.: Interpretation of (U–Th)/He single grain ages from slowly cooled crustal terranes: A case study from the Transantarctic Mountains of southern Victoria Land, *Chem. Geol.*, 225, 91–120, <https://doi.org/10.1016/j.chemgeo.2005.09.001>, 2006.
- 385 Flowers, R. M., Ketcham, R. A., Shuster, D. L., and Farley, K. A.: Apatite (U–Th)/He thermochronometry using a radiation damage accumulation and annealing model, *Geochim. Cosmochim. Acta*, 73, 2347–2365, <https://doi.org/10.1016/j.gca.2009.01.015>, 2009.
- 390 Flowers, R. M., Zeitler, P. K., Danišik, M., Reiners, P. W., Gautheron, C., Ketcham, R. A., Metcalf, J. R., Stockli, D. F., Enkelmann, E., and Brown, R. W.: (U–Th)/He chronology: Part 1. Data, uncertainty, and reporting, *Geol. Soc. Am. Bull.*, 2022a. In press.
- Flowers, R. M., Ketcham, R. A., Enkelmann, E., Gautheron, C., Reiners, P. W., Metcalf, J. R., Danišik, M., Stockli, D. F., and Brown, R. W.: (U–Th)/He chronology: Part 2. Considerations for evaluating, integrating, and interpreting conventional individual aliquot data, *Geol. Soc. Am. Bull.*, 2022b. In press.
- 395



- Gautheron, C., Tassan-Got, L., Ketcham, R. A., and Dobson, K. J.: Accounting for long alpha-particle stopping distances in (U-Th-Sm)/He geochronology: 3D modeling of diffusion, zoning, implantation, and abrasion, *Geochim. Cosmochim. Acta*, 96, 44–56, <https://doi.org/10.1016/j.gca.2012.08.016>, 2012.
- 400 He, J., Stuart N. Thomson, and Peter W. Reiners.: Apatite (U-Th)/He thermochronometric data show rapid late Eocene incision in the central Transantarctic Mountains, *AGU Fall Meeting Abstracts*, vol. 2019, C53B-1340, <https://ui.adsabs.harvard.edu/abs/2019AGUFM.C53B1340H/abstract>, 2019.
- He, J., Thomson, S. N., Reiners, P. W., Hemming, S. R., and Licht, K. J.: Rapid erosion of the central Transantarctic Mountains at the Eocene-Oligocene transition: Evidence from skewed (U-Th)/He date distributions near Beardmore Glacier, *Earth Planet. Sci. Lett.*, 567, 117009, <https://doi.org/10.1016/j.epsl.2021.117009>, 2021.
- 405 Herman, F., Braun, J., Senden, T. J., and Dunlap, W. J.: (U-Th)/He thermochronometry: Mapping 3D geometry using micro-X-ray tomography and solving the associated production-diffusion equation, *Chem. Geol.*, 242, 126–136, <https://doi.org/10.1016/j.chemgeo.2007.03.009>, 2007.
- Hourigan, J. K., Reiners, P. W., and Brandon, M. T.: U-Th zonation-dependent alpha-ejection in (U-Th)/He chronometry, *Geochim. Cosmochim. Acta*, 69, 3349–3365, <https://doi.org/10.1016/j.gca.2005.01.024>, 2005.
- 410 Ketcham, R. A., Gautheron, C., and Tassan-Got, L.: Accounting for long alpha-particle stopping distances in (U-Th-Sm)/He geochronology: Refinement of the baseline case, *Geochim. Cosmochim. Acta*, 75, 7779–7791, <https://doi.org/10.1016/j.gca.2011.10.011>, 2011.
- Meesters, A. G. C. A. and Dunai, T. J.: Solving the production-diffusion equation for finite diffusion domains of various shapes: Part I. Implications for low-temperature (U-Th)/He thermochronology, *Chem. Geol.*, 186, 333–344, 2002.
- 415 Murray, K. E., Orme, D. A., and Reiners, P. W.: Effects of U-Th-rich grain boundary phases on apatite helium ages, *Chem. Geol.*, 390, 135–151, <https://doi.org/10.1016/j.chemgeo.2014.09.023>, 2014.
- Palache, C., Clifford, F., and Berman, H.: *The System of Mineralogy*, John Wiley and Sons, 1963.
- Reiners, P. W. and Farley, K. A.: Influence of crystal size on apatite (U-Th)/He thermochronology: an example from the Bighorn Mountains, Wyoming, *Earth Planet. Sci. Lett.*, 188, 413–420, [https://doi.org/10.1016/S0012-821X\(01\)00341-7](https://doi.org/10.1016/S0012-821X(01)00341-7),
420 2001.
- Reiners, P. W., Thomson, S. N., McPhillips, D., Donelick, R. A., and Roering, J. J.: Wildfire thermochronology and the fate and transport of apatite in hillslope and fluvial environments, *J. Geophys. Res. Earth Surf.*, 112, <https://doi.org/10.1029/2007JF000759>, 2007.
- Reiners, P. W., Carlson, R. W., Renne, P. R., Cooper, K. M., Granger, D. E., McLean, N. M., and Schoene, B.:
425 *Geochronology and Thermochronology*, Wiley, 2018.
- Spiegel, C., Kohn, B., Belton, D., Berner, Z., and Gleadow, A.: Apatite (U-Th-Sm)/He thermochronology of rapidly cooled samples: The effect of He implantation, *Earth Planet. Sci. Lett.*, 285, 105–114, <https://doi.org/10.1016/j.epsl.2009.05.045>, 2009.
- 430 Thomson, Stuart N., Peter W. Reiners, John He, Sidney R. Hemming, and Kathy Licht: New constraints on the pre-glacial and glacial uplift and incision history of the central Transantarctic Mountains using multiple low-temperature thermochronometers, *AGU Fall Meeting Abstracts*, Vol. 2019, C14B-04, <https://ui.adsabs.harvard.edu/abs/2019AGUFM.C14B.04T/abstract>, 2019.



Whipp, D. M., Kellett, D. A., Coutand, I., and Ketcham, R. A.: Short communication: Modeling competing effects of cooling rate, grain size, and radiation damage in low-temperature thermochronometers, 4, 143–152, 435 <https://doi.org/10.5194/gchron-4-143-2022>, 2022.

Wolf, R. A., Farley, K. A., and Kass, D. M.: Modeling of the temperature sensitivity of the apatite (U-Th)/He thermochronometer, *Chem. Geol.*, 148, 105–114, [https://doi.org/10.1016/S0009-2541\(98\)00024-2](https://doi.org/10.1016/S0009-2541(98)00024-2), 1998.

Zeitler, P. K., Herczeg, A. L., McDougall, I., and Honda, M.: U-Th-He dating of apatite: A potential thermochronometer, *Geochim. Cosmochim. Acta*, 51, 2865–2868, [https://doi.org/10.1016/0016-7037\(87\)90164-5](https://doi.org/10.1016/0016-7037(87)90164-5), 1987.

440 Ziegler, J. F.: Helium: Stopping powers and ranges in all elemental matter., 4, 1977.